Use of A PC-Based Data Acquisition System for Measurement of Complex Mechanical Resonances/Vibrations of Musical Instruments

For the UIUC Physics 199POM/Physics 498POM courses, we have developed a PC-based data acquisition (DAQ) system for the purpose of measuring the so-called *complex* (i.e. phase-sensitive) mechanical vibrations of various kinds of musical instruments as a function of frequency. As shown below in the block diagram of this experimental setup, the PC is used to set the amplitude of, and step the frequency of an Agilent sine-wave function generator over a user-defined frequency range $f_{\min} \le f \le f_{\max}$, usually in the audio band. In order to achieve a high signal-to-noise ratio, the AC sine-wave signal output from the Agilent function generator (which has a maximum amplitude of 5.0 Volts) is input to a custom-built high-voltage amplifier that has a voltage gain of 10x. The 10x-amplified AC sine-wave signal output from the high-voltage amplifier is then used to drive a wafer-thin 1" diameter piezo-electric transducer, which due to the nature of the piezo-electric material converts the electrical energy into mechanical vibrational energy at the driving frequency f of the function generator. Thus, the "transmitter" piezo-electric transducer acts like a miniature loudspeaker, injecting mechanical energy into a musical instrument, e.g. a guitar, placed in proximity to the bridge of the guitar in order to mimic as closely as possible the natural mechanical vibrations of the bridge when the guitar is played normally.

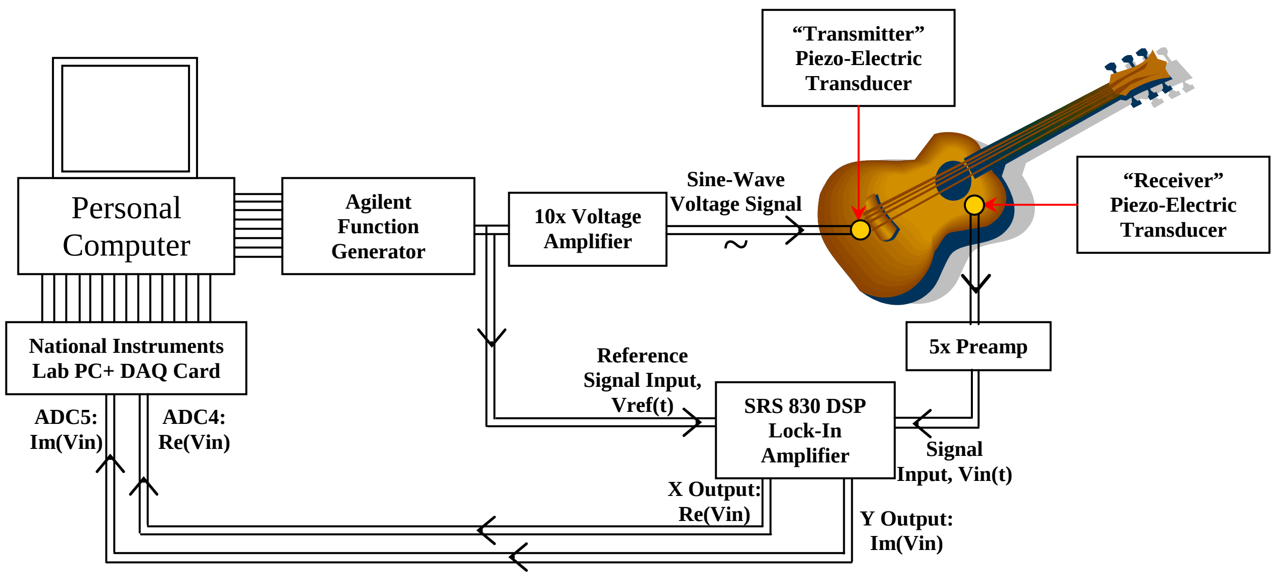

Figure 1: Setup for Sonic PC-Based DAQ Experiment

A second wafer-thin "receiver" piezo-electric transducer, used as a "microphone", is placed at any other interesting/ convenient location on the musical instrument in order to detect the mechanical vibrations present at that location caused by the mechanical vibrations from the driving force(s) input from the "transmitter" piezo-electric transducer e.g. located at the bridge of a guitar. The AC signal output from the "receiver" piezo-electric transducer is amplified 5x via a custom-built low-noise preamplifier instrumentation op-amp circuit, which has high input impedance and low output impedance, which is input to a Stanford Research Systems SRS-830 DSP-based lock-in amplifier. The AC sine-wave signal output from the Agilent function generator $V_{FG}(t) = V_o^{FG} \sin \omega t$ is also sent to the SRS-830 lock-in amplifier, and is used as a *reference* signal. The lock-in amplifier compares this AC sine-wave reference signal $V_{FG}(t) = V_o^{FG} \sin \omega t$ to the AC sine-wave signal output from the "receiver" piezo-electric transducer $V_{Pzo}^{Rcvr}(t) = V_o^{Rcvr\ Pzo} \sin[\omega t + \varphi(\omega)]$, where $\varphi(\omega)$ is a {frequency-dependent} phase shift; $\omega = 2\pi f$ (radians/second) is the so-called angular frequency and *f* = the frequency of oscillation (Hz, or cycles per second). An example of these two waveforms is shown in Figure 2 below – for numerical values of $V_o^{FG} = 1.4\,Volts$, $V_o^{Rcvr\ Pzo} = 1.0\,Volts$, $\omega = 10\,radians/second$ and $\varphi(\omega) = \pm 0.57\,radians$.

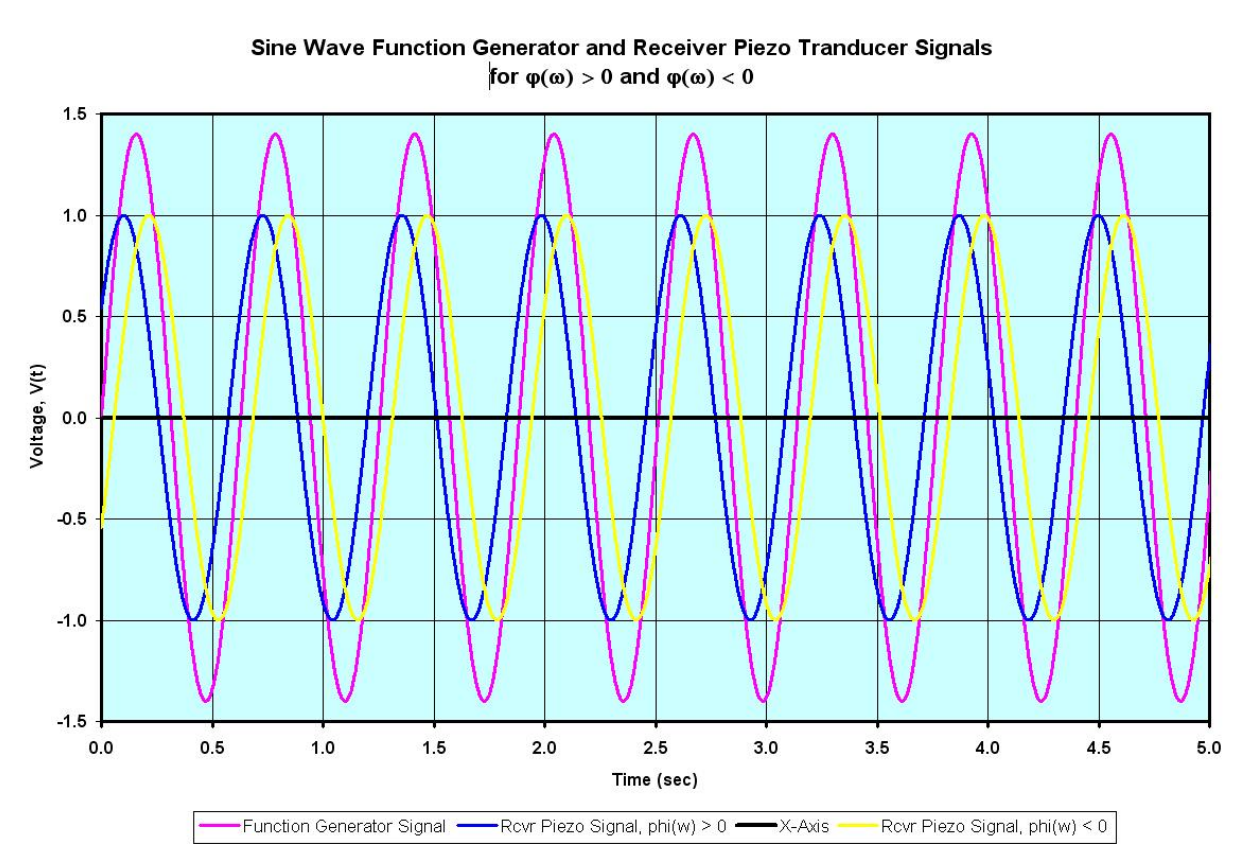

Figure 2: Waveforms associated with a.) the Agilent sine-wave function generator (input stimulus to the musical instrument) and b.) the "receiver" piezo-electric transducer (output response from the musical instrument).

Note that in the above figure, the $\varphi(\omega) > 0$ $(\varphi(\omega) < 0)$ waveform of the "receiver" piezo-electric transducer is respectively shifted earlier (later) in time relative to the sine-wave function generator waveform – in this situation we say that the "receiver" piezo-electric transducer signal leads (lags) the sine-wave function generator signal by the amount $\varphi(\omega) > 0$ $(\varphi(\omega) < 0)$ radians.

Using the trigonometric identity $\sin[A + B] = \sin A \cos B + \cos A \sin B$ we see that we can write:

$$V_{Pzo}^{Rcvr}(t) = V_o^{Rcvr\ Pzo} \sin[\omega t + \varphi(\omega)] = V_o^{Rcvr\ Pzo} \sin \omega t \cos \varphi(\omega) + \cos \omega t \sin \varphi(\omega)$$

This relation can also be rewritten in the following form as:

$$V_{Pzo}^{Rcvr}(t) = \left[V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right]\sin\omega t + \left[V_o^{Rcvr\ Pzo}\sin\varphi(\omega)\right]\cos\omega t$$

The first term in the above expression can be seen to be in-phase with the driving signal from the Agilent function generator $V_{FG}(t) = V_o^{FG}\sin\omega t$ whereas the second term is 90^o out-of-phase with the driving signal from the Agilent function generator.

The SRS-830 DSP lock-in amplifier is a truly miraculous, highly sophisticated and extremely versatile electronic device. Since the lock-in amplifier uses the sine-wave signal output from the Agilent function generator as its reference signal, it knows precisely what frequency *f* to be looking for in the input signal, even if the input signal is noisy (n.b. the noise can be at any frequency) - and if much of the noise is not at the reference frequency *f*, it is almost completely ignored by the lock-in amplifier! The digital signal processor (DSP) inside the guts of the lock-in amplifier carries out some fancy mathematical calculations to figure out how much of the input signal from the "receiver" piezo-electric transducer is in-phase with the reference signal $\left[V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right]$ and how much of the input signal is 90^o out-of-phase with the reference signal, the DSP then outputs two DC voltages that are ***proportional*** to the in-phase and 90^o out-of-phase components of the input signal from the "receiver" piezo-electric transducer!

We can *graphically* represent the in-phase and 90^o out-of-phase DC voltage components output from the lock-in amplifier that are associated with the AC signal from the "receiver" piezo-electric transducer as x- and y-components in a two-dimensional graph known as the complex plane, with the horizontal/x-axis representing the in-phase component $\left[V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right]$ and the vertical/y-axis representing the 90^o out-of-phase component $\left[V_o^{Rcvr\ Pzo}\sin\varphi(\omega)\right]$, as shown in Figure 3 below:

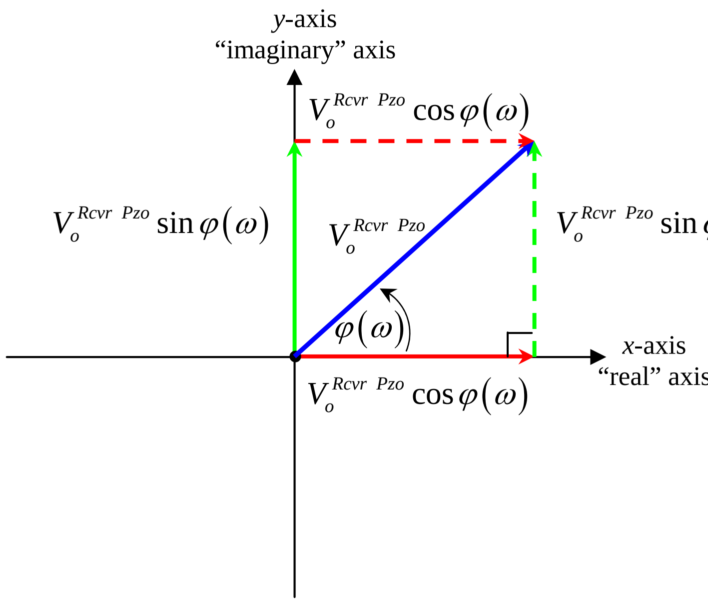

Figure 3: The complex plane

From the right triangle in the above figure and use of Pythagoras' theorem $hyp = \sqrt{adj^2 + opp^2}$ (i.e. $r = \sqrt{x^2 + y^2}$) we see that the <u>magnitude</u> (i.e. the length) of the hypotenuse is given by:

$$V_o^{Rcvr\ Pzo} = \sqrt{\left[V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right]^2 + \left[V_o^{Rcvr\ Pzo}\sin\varphi(\omega)\right]^2} = V_o^{Rcvr\ Pzo}\sqrt{\cos^2\varphi(\omega) + \sin^2\varphi(\omega)} = V_o^{Rcvr\ Pzo}$$

We also see from the above figure that:

$$\varphi(\omega) = \tan^{-1}\left(V_o^{Rcvr\ Pzo}\sin\varphi(\omega)/V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right) = \tan^{-1}\left(\sin\varphi(\omega)/\cos\varphi(\omega)\right) = \tan^{-1}\left(\tan\varphi(\omega)\right) = \varphi(\omega)$$

In the jargon of mathematicians and physicists, the <u>in-phase</u> component of an <u>*arbitrary*</u> complex signal $\tilde{V}(t)$ is known as the <u>real part</u> of $\tilde{V}(t)$ and is denoted as $\Re e\{\tilde{V}(t)\}$; the 90^o <u>out-of-phase</u> component of the complex signal $\tilde{V}(t)$ is known as the <u>imaginary part</u> of $\tilde{V}(t)$ and is denoted as $\Im m\{\tilde{V}(t)\}$. An arbitrary complex signal can thus be represented, in general as: $\tilde{V}(t) \equiv \Re e\{\tilde{V}(t)\} + i\,\Im m\{\tilde{V}(t)\}$ where $i \equiv \sqrt{-1}$. The *i* is there to remind us that this component is 90^o out of phase with the reference signal. The <u>magnitude</u> of complex $\tilde{V}(t)$ is denoted as $\left|\tilde{V}(t)\right| = \sqrt{\tilde{V}(t)\tilde{V}^*(t)} = \sqrt{\Re e\{\tilde{V}(t)\}^2 + \Im m\{\tilde{V}(t)\}^2}$ where $\tilde{V}^*(t)$ is the so-called complex conjugate of $\tilde{V}(t)$ and is defined as $\tilde{V}^*(t) \equiv \Re e\{\tilde{V}(t)\} - i\,\Im m\{\tilde{V}(t)\}$ and $-i = -\sqrt{-1}$ {thus, $-i\cdot i = -\sqrt{-1}\sqrt{-1} = -(-1) = +1$}. The phase angle is defined as $\varphi \equiv \tan^{-1}\left(\Im m\{\tilde{V}(t)\}/\Re e\{\tilde{V}(t)\}\right)$.

Note also that in the above figure, if the real, or in-phase component $V_o^{Rcvr\ Pzo}\cos\varphi(\omega) < 0$, the minus sign physically means that this component of the signal is 180^o out-of-phase with the reference signal. Similarly, if the imaginary, or 90^o out-of phase component $V_o^{Rcvr\ Pzo}\sin\varphi(\omega) < 0$ the minus sign here physically means that this component of the signal is –90^o out-of-phase with the reference signal, as opposed to being +90^o out-of-phase, if $V_o^{Rcvr\ Pzo}\sin\varphi(\omega) > 0$.

The DC voltages output from the lock-in amplifier that are proportional to the <u>in-phase</u> component $\left[V_o^{Rcvr\ Pzo}\cos\varphi(\omega)\right]$ and 90^o <u>out-of-phase</u> component $\left[V_o^{Rcvr\ Pzo}\sin\varphi(\omega)\right]$ of the AC signal "receiver" piezo-electric transducer are then each input to an ADC on a National Instruments LabPC+ DAQ card. What is an ADC? An ADC is an acronym for <u>A</u>nalog-to-<u>D</u>igital <u>C</u>onverter. An ADC is another very useful circuit that converts a DC voltage into a digital number. The National Instruments LabPC+ DAQ card has 8 ADC channels, however for the Sonic Mechanical Resonance DAQ setup, we only need to use two of the eight ADC channels, in order to digitize the two DC voltages output from the lock-in amplifier. The ADC's on the National Instruments LabPC+ DAQ card have 12 bits of digital resolution spanning an input voltage range of $\pm 5.0\,Volts$. This means that any DC voltage input to a 12-bit ADC within this 10.0 Volt span can be represented by a binary number (i.e. 0's and 1's) ranging from a lowest value of 000000000000b (= 0 decimal) to its highest value of 111111111111b (= 4095 decimal).

For 12-bit ADC's, there are $2^{12} = 4096$ (decimal) possible distinct numbers (0,1,2,3,4,… ….4093,4094,4095). Thus, a 12-bit ADC "quantizes" a continuum of DC voltages between $\pm 5.0\,Volts$ to 4096 values between these limits, with 10V/4096 = 0.00244 Volt least-significant-bit resolution. For a high-quality/well-designed ADC, the relationship between DC voltage input vs. digital number output is nearly perfectly linear, as shown in Figure 4 below:

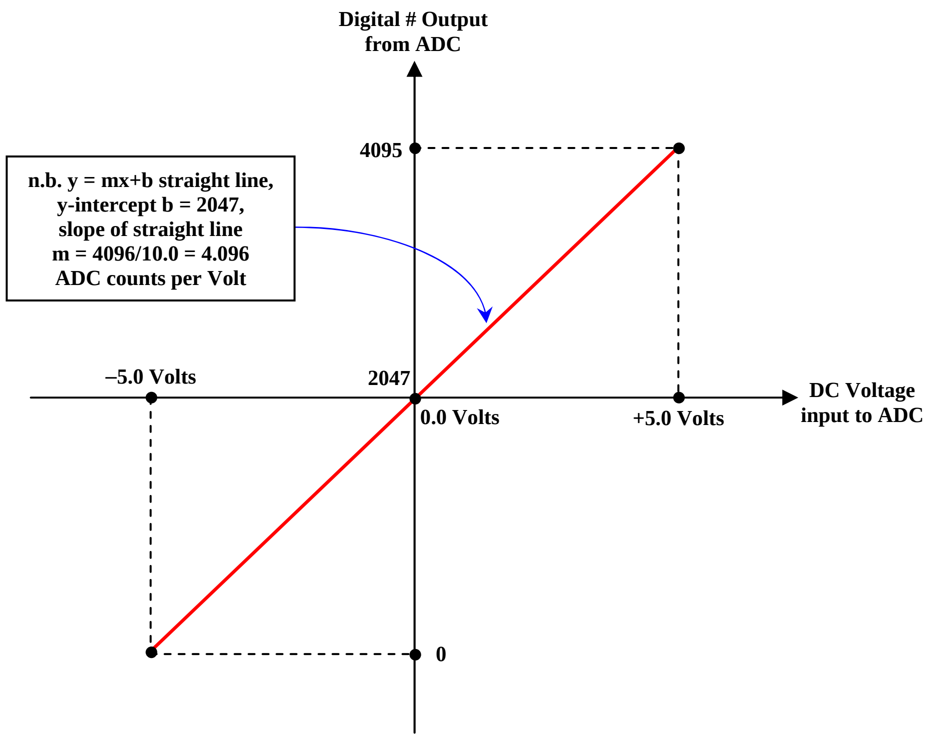

Figure 4. The linear relationship between ADC value vs. DC voltage.

Note that we also employ the use of *signal-averaging techniques* in the Sonic DAQ program in order to minimize the effects of (relatively small amounts of) electrical noise fluctuations and electromagnetic noise pickup associated with the ADC measurements of the DC voltages output from the lock-in amplifier. For *each* frequency setting of the Agilent function generator f, we (rapidly) take a total of 5000 ADC measurements for each ADC and then compute the average, or mean ADC voltage value <V_ADC> for each of the two ADC channels.

It turns out that when we have *no* signal present from the Agilent function generator, we find that the <V_ADC's> are NOT precisely equal to 0.000 volts, but something very slightly different from zero, typically on the order of a fraction of a milli-Volt, tiny in comparison to 5.0 Volts. These slight DC voltage offsets associated with each of the ADC channels occur for a number of reasons – the electronic circuitry in each of the ADC channels isn't perfect – internally, each ADC circuit often has a slight DC voltage offset. Also, for no signal present at the input of the lock-in amplifier, the two DC voltages output from the lock-in amplifier aren't perfectly equal to 0.0000 volts either. Furthermore, there can exist very small contact potential differences between different metals – e.g. chrome-plated BNC connectors and gold-plated pins, copper pc board traces, solder, etc. on the pc boards inside the lock-in amplifier, etc. Periodically, we take no-signal Sonic DAQ runs in order to explicitly measure these no-signal <V_ADC> DC voltage offsets. We call these no-signal DC voltage offsets in the <V_ADC> values "pedestals". These "pedestals" are then removed from the raw <V_ADC> data by subtracting them for each ADC:

$$\langle \text{V\_ADC} \rangle_{true} = \langle \text{V\_ADC} \rangle_{raw} - \langle \text{V\_ADC} \rangle_{ped}$$

At each frequency setting *f* of the Agilent function generator (over the user-defined range $f_{\min} \le f \le f_{\max}$) the Sonic DAQ program explicitly calculates:

$$\boxed{\Re e\left(V_{in}\left(f\right)\right) \equiv \langle \text{V\_ADC4}(f) \rangle_{true} = \langle \text{V\_ADC4}(f) \rangle_{raw} - \langle \text{V\_ADC4} \rangle_{ped}}$$

$$\boxed{\Im m\left(V_{in}\left(f\right)\right) \equiv \langle \text{V\_ADC5}(f) \rangle_{true} = \langle \text{V\_ADC5}(f) \rangle_{raw} - \langle \text{V\_ADC5} \rangle_{ped}}$$

At each frequency setting *f* of the Agilent function generator, the Sonic DAQ program then subsequently calculates the magnitude of $V_{in}(f)$ and the phase angle:

$$\boxed{\left|V_{in}(f)\right| \equiv \sqrt{\Re e\left(V_{in}(f)\right)^2 + \Im m\left(V_{in}(f)\right)^2}} \quad \text{and} \quad \boxed{\varphi(f) = \tan^{-1}\left(\frac{\Im m\left(V_{in}(f)\right)}{\Re e\left(V_{in}(f)\right)}\right)}$$

The Sonic DAQ program then stores the raw ADC data $\langle \text{V\_ADC4}(f) \rangle_{raw}, \langle \text{V\_ADC5}(f) \rangle_{raw}$ and the pedestal-corrected ADC data $\Re e\left(V_{in}(f)\right) \equiv \langle \text{V\_ADC4}(f) \rangle_{true}$, $\Im m\left(V_{in}(f)\right) \equiv \langle \text{V\_ADC5}(f) \rangle_{true}$ as well as the calculated data $\left|V_{in}(f)\right| \equiv \sqrt{\Re e\left(V_{in}(f)\right)^2 + \Im m\left(V_{in}(f)\right)^2}$, $\varphi(f) = \tan^{-1}\left(\frac{\Im m\left(V_{in}(f)\right)}{\Re e\left(V_{in}(f)\right)}\right)$ in individual arrays, which can then be written out to a datafile at the end of data-taking and e.g. can also be viewed in on-line plots of each of these variables.

Once the Sonic DAQ program has completed the above operations and computations at each frequency setting *f* of the Agilent function generator, the Sonic DAQ program then issues a command to the Agilent function generator to increase the frequency setting by a user-defined amount f_{step}, which is typically f_{step} = 1.0 Hz for fine-resolution "slow" scans, e.g. over the user-defined frequency range, which is typically from ~ 20 Hz to 1020 Hz, or, sometimes f_{step} = 10 Hz is used for coarse-resolution "fast" scans over this same frequency range, or e.g. for scans over the full audio frequency range of ~ 20 Hz to 20 KHz.

Each time the frequency setting of the Agilent sine-wave function generator is increased, the Sonic DAQ program pauses doing anything for several seconds (nominally a 5 second wait) in order to allow time for the DC voltages at the two outputs of the SRS 830 DSP lock-in amplifier to {asymptotically} settle to their final values at the new frequency setting. The Sonic DAQ program then obtains the new $\langle \text{V\_ADC} \rangle_{raw}$ values, computes the new $\Re e\left(V_{in}(f)\right)$, $\Im m\left(V_{in}(f)\right)$, $\left|V_{in}(f)\right|$ and $\varphi(f)$, stores these in their respective arrays, and then the whole process repeats over and over, until the last measurements at the highest frequency setting $f_{\max}$ of the Agilent sine-wave function generator have been obtained. Upon completion of the data-taking with the Sonic DAQ program, as mentioned above, via pull-down menus, has on-line plots of the raw data as well as the pedestal-corrected data and all of the other computed variables, this data can also can be written out to a (text-based) datafile, for further "off-line" analysis.