Physics 435
Spring 2008

Homework #6

1) Write the first 2 non-vanishing terms for the $r > R$ potential due to a charge distribution of the form $\sigma = +k$ for $0 < \theta < \pi/2$ and $\sigma = -k$ for $\pi/2 < \theta < \pi$

2) Recall the problem of a charge distribution of the form $\sigma = k\cos^2\theta$ glued to an insulating spherical shell of radius R discussed in lecture. In this problem we will compute the electrostatic pressure on an infinitesimal patch of charge on the "north pole" at $\theta = 0$ in two ways. Feel free to use any an all of the results obtained in lecture to help you. Hint-in computing the force on a patch of charge it is important to remove the field due to the charge itself.

   a) Compute the pressure using the $V(r > R) = \sum_{\ell=0,1,2} \frac{B_\ell}{r^{\ell+1}} \mathrm{P}_\ell(\cos\theta)$ expression.

   b) Check your result by computing the pressure using the $V(r < R) = \sum_{\ell=0,1,2} A_\ell r^\ell \mathrm{P}_\ell(\cos\theta)$ expression.

3) Verify that $\vec{E}(\vec{r}) = \frac{3(\vec{p}\bullet\hat{r})\hat{r} - \vec{p}}{4\pi\varepsilon_0 r^3}$ starting from

$$E(x,y,z) = -\frac{1}{4\pi\varepsilon_0}\vec{\nabla}\frac{xp_x + yp_y + zp_z}{\left(x^2 + y^2 + z^2\right)^{3/2}}$$

   a) Check your result for the case where $\vec{r} = z\,\hat{z}$ and the dipole consists of A positive charge at $\vec{r}\,' = b\hat{z}$ and an equal and opposite charge at $\vec{r}\,' = -b\hat{z}$ directly from Coulomb's Law. Work in the appropriate limit: $z \gg b$.

4) Griffiths Problem 3.23. Hints: The azimuthal part of the solutions is sinusoidal with an integral constant of separation to insure that $\Phi(\phi + 2\pi) = \Phi(\phi)$. The comment concerning the infinite line solutions with $S \propto \ln s$ concerns is relevant for the case where $\Phi(\phi)$ is a constant. Try a power law solution for *S(s)* associated with the other sinusoidal $\Phi(\phi)$ possibilities.

5) Consider a charge distribution of the form $\rho = \rho_0\, z \exp\left(-\beta\left[x^2 + y^2 + z^2\right]\right)$. Find the dipole moment $\vec{p} = \int \rho\, \vec{r}\,'\, d\tau'$ and use it to obtain an approximate expression for the potential far from the origin. Hint—this integral is useful $\int_0^\infty x^n \exp(-x)\, dx = n!$