Questions for P435 Lecture Notes 4

1.) Work & Electrostatic Energy: In physically moving a test charge q_T from point *a* to point *b* along the path $a \to b$, why is the mechanical force $\vec{F}_{mech}(\vec{r})$ equal to the *negative* of the electrostatic force $\vec{F}_C(\vec{r})$ acting on the test charge at field point(s) $\vec{r}$?

2.) The mechanical work done *W* on a test charge q_T in physically moving it from point *a* to point *b* along the path $a \to b$ is equal to the *change* in the electrostatic potential energy between points *a* and *b*, i.e. $W = P.E.(r = b) - P.E.(r = a)$

If the reference point (*a*) is taken at $r = \infty$ (i.e. the zero of potential energy, $P.E.(r = \infty) = 0$), then we say that the mechanical work done *W* in physically moving the test charge from $r = \infty$ to $r = b$ is equal to the potential energy at $W = P.E.(r = b)$.

3.) For the 4 boxed formulas given for W_{TOT} on page 8 of P435 lecture notes 4, can you derive the last three formulas yourself?

4.) Does it make physical sense that the electrostatic energy associated with e.g. an electron is truly infinite, especially since the electron has a finite mass, $m_e = 0.511 MeV/c^2$?
{n.b. We discussed whether Coulomb's force law gave a truly infinite attractive/repulsive force for $r = 0$ at the beginning of P435 lecture on Monday, 8/27/07}.

5.) Why does the mechanical work needed to assemble an electrostatic charge distribution *not* obey the principle of linear superposition?

Answers to Questions for P435 Lecture Notes 4

1.) Work & Electrostatic Energy: In physically moving a test charge q_T from point a to point b along the path $a \to b$, why is the mechanical force $\vec{F}_{mech}(\vec{r})$ equal to the *negative* of the electrostatic force $\vec{F}_C(\vec{r})$ acting on the test charge at field point(s) $\vec{r}$?

$\vec{F}_{mech}(\vec{r}) = -F_C(\vec{r})$ by Newton's 3rd law of {Galilean} motion – here, the mechanical force $\vec{F}_{mech}(\vec{r})$ is required to balance/cancel the electrostatic Coulomb force $\vec{F}_C(\vec{r})$ acting on the test charge q_T. at the field/observation point $\vec{r}$ due to the {source} charge q (located at the local origin.)

2.) The mechanical work done W on a test charge q_T in physically moving it from point a to point b along the path $a \to b$ is equal to the *change* in the electrostatic potential energy between points a and b, i.e. $W = P.E.(r=b) - P.E.(r=a)$

If the reference point (a) is taken at $r = \infty$ (i.e. the zero of potential energy, $P.E.(r=\infty) = 0$), then we say that the mechanical work done W in physically moving the test charge from $r = \infty$ to $r = b$ is equal to the potential energy at $W = P.E.(r=b)$.

3.) For the 4 boxed formulas given for W_{TOT} on page 8 of P435 lecture notes 4, can you derive the last three formulas yourself?

Please see/read/work through the P435 Lecture Notes 4, p 8. Note that the work increment done on an infinitesimal amount of charge is $dW(\vec{r}) = dq(\vec{r})V(\vec{r})$ and $dq(\vec{r}) = \rho(\vec{r})d\tau$, for volume charge densities, or: $dq(\vec{r}) = \sigma(\vec{r})dA$ for surface charge densities, or: $dq(\vec{r}) = \lambda(\vec{r})d\ell$ for line charge densities.

4.) Does it make physical sense that the electrostatic energy associated with e.g. an electron is truly infinite, especially since the electron has a finite mass, $m_e = 0.511 MeV/c^2$?
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Please see/read "Answers to Questions for P435 Lecture Notes # 1", Question/Answer # 1. It can then be seen that we shouldn't be upset that $U(r \equiv 0) = \infty$, because we know that extrapolating classical mean-field results associated with finite r to $r \equiv 0$ is bound to fail/will fail/does fail because other physics becomes operative long before $r \equiv 0$ is reached!

5.) Why does the mechanical work W needed to assemble an electrostatic charge distribution *not* obey the principle of linear superposition?

Because W (a scalar quantity) is proportional to the modulus *squared* of the total electric field, and thus e.g. if $\vec{E}_{Tot}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r})$ then:

$$\vec{E}_{Tot}(\vec{r}) \bullet \vec{E}_{Tot}(\vec{r}) = \left[\vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r})\right] \bullet \left[\vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r})\right]$$

$$= \vec{E}_1^2(\vec{r}) + 2\vec{E}_1(\vec{r}) \bullet \vec{E}_2(\vec{r}) + \vec{E}_2^2(\vec{r})$$

The quantity $2\vec{E}_1(\vec{r}) \bullet \vec{E}_2(\vec{r})$ is formally known as an *interference* term (!) thus, we see that the electric field is actually behaving as an amplitude, rather than an intensity!!!