Questions for P435 Lecture Notes 3

1.) What two mathematical conditions *uniquely* specifies the nature of an arbitrary, but differentiable vector field $\vec{F}(\vec{r})$ that goes to zero faster than 1/*r* as $r \to \infty$.

Answer: $\vec{\nabla}\bullet\vec{F}(\vec{r})$ and $\vec{\nabla}\times\vec{F}(\vec{r})$. Why???

See/read Griffiths Appendix B *r.e.* the Helmholtz theorem, and in particular, the ***corollary*** to the Helmholtz theorem, p. 557.

2.) Is the *absolute* potential $V(\vec{r})$ at an arbitrary point in space $(\vec{r})$ a physically meaningful quantity? Or are only potential differences $\Delta V_{ab} \equiv V(r_b) - V(r_a)$ physically meaningful?

3.) Know/understand the concept of **equipotential** surfaces. Especially note/understand that contours of constant potential are perpendicular to electric field lines $\vec{E}(\vec{r})$ everywhere in space!

4.) Know/understand the relation between electric field $\vec{E}(\vec{r})$ and the scalar potential $V(\vec{r})$, namely $\vec{E}(\vec{r}) = -\nabla V(\vec{r})$ (in differential form)

and $\Delta V_{ab} \equiv V(r_b) - V(r_a) = -\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell}$ (in integral form).

5.) Know/understand how Poisson's equation $\nabla^2 V(\vec{r}) = -\rho_{encl}/\varepsilon_o$ and Laplace's equation $\nabla^2 V(\vec{r}) = 0$ are obtained by using the relations $\vec{\nabla}\bullet\vec{E}(\vec{r})$, $\vec{\nabla}\times\vec{E}(\vec{r})$ and $\vec{E}(\vec{r}) = -\nabla V(\vec{r})$.

6.) How is the boundary condition $E_{above}^{\perp} - E_{below}^{\perp} = \sigma/\varepsilon_0$ obtained?
What is the physical meaning of this boundary condition?

7.) How is the boundary condition $E_{above}^{\parallel} - E_{below}^{\parallel} = 0$ obtained?
What is the physical meaning of this boundary condition?

8.) How is the boundary condition $\left.\frac{\partial V_{above}}{\partial n}\right|_{interface} - \left.\frac{\partial V_{below}}{\partial n}\right|_{interface} = -\left(\frac{\sigma}{\varepsilon_0}\right)$ obtained?
What is the physical meaning of this boundary condition?

Answers to Questions for P435 Lecture Notes 3

1.) What two mathematical conditions *uniquely* specifies the nature of an arbitrary, but differentiable vector field $\vec{F}(\vec{r})$ that goes to zero faster than $1/r$ as $r \to \infty$.

Answer: $\vec{\nabla} \bullet \vec{F}(\vec{r})$ and $\vec{\nabla} \times \vec{F}(\vec{r})$ fully specify the physics nature of the vector field $\vec{F}(\vec{r})$ for the class of differentiable vector functions $\vec{F}(\vec{r})$ that go to zero faster than $1/r$ as $r \to \infty$. Please see/read Griffiths Appendix B *r.e.* the Helmholtz theorem, and in particular, the ***corollary*** to the Helmholtz theorem, p. 557.

2.) Is the *absolute* potential $V(\vec{r})$ at an arbitrary point in space $(\vec{r})$ a physically meaningful quantity? Or are only potential *differences* $\Delta V_{ab} \equiv V(r_b) - V(r_a)$ physically meaningful?

Indeed, only *differences* in potential $\Delta V_{ab} \equiv V(r_b) - V(r_a)$ are physically meaningful; the *absolute* potential $V(\vec{r})$ at an arbitrary point in space $(\vec{r})$ has no physical meaning.

This aspect of the (scalar) potential $V(\vec{r})$ actually goes quite deep, into the heart of electromagnetism, for it is intimately connected to the so-called gauge invariant nature of the electromagnetic interaction. One can always add an arbitrary constant to the potential $V(\vec{r})$ and this will/can have no physically observable consequences. We will also see/learn later on in the P436 course that one can also add an arbitrary gradient of a scalar function $\vec{\nabla}\lambda(\vec{r})$ to the (so-called magnetic) vector potential $\vec{A}(\vec{r})$ also with no physically observable consequences – please see/read P436 Lecture Notes 16. The scalar potential $V(\vec{r})$ and the vector potential $\vec{A}(\vec{r})$ respectively are the temporal and spatial components of the relativistic four-vector potential $A^{\mu}(\vec{r}) \equiv \left(V(\vec{r})/c, \vec{A}(\vec{r})\right) = \left(A^0, A^1, A^2, A^3\right) = \left(A^0, A_x, A_y, A_z\right)$.

3.) Know/understand the concept of **equipotential** surfaces. Especially note/understand that contours of constant potential are perpendicular to electric field lines $\vec{E}(\vec{r})$ everywhere in space!

Equipotential surfaces are everywhere perpendicular to electric field lines $\vec{E}(\vec{r})$ because of / due to the fact that $\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$.

4.) Know/understand the relation between electric field $\vec{E}(\vec{r})$ and the scalar potential $V(\vec{r})$, namely $\vec{E}(\vec{r}) = -\nabla V(\vec{r})$ (in differential form)

and $\Delta V_{ab} \equiv V(r_b) - V(r_a) = -\int_a^b \vec{E}(\vec{r}) \bullet d\vec{\ell}$ (in integral form).

Please see/read P435 Lecture Notes 3, p. 2-3.

5.) Know/understand how Poisson's equation $\nabla^2 V(\vec{r}) = -\rho_{encl}/\varepsilon_o$ and Laplace's equation $\nabla^2 V(\vec{r}) = 0$ are obtained by using the relations $\vec{\nabla}\cdot\vec{E}(\vec{r})$, $\vec{\nabla}\times\vec{E}(\vec{r})$ and $\vec{E}(\vec{r}) = -\nabla V(\vec{r})$.

Please see/read P435 Lecture Notes 3, p. 15-16.

6.) How is the boundary condition $E^{\perp}_{above} - E^{\perp}_{below} = \sigma/\varepsilon_0$ obtained?
What is the physical meaning of this boundary condition?

Use Gauss' law – please see/read P435 Lecture Notes 3, p. 17.

7.) How is the boundary condition $E^{\parallel}_{above} - E^{\parallel}_{below} = 0$ obtained?
What is the physical meaning of this boundary condition?

This actually arises from the integral form of $\vec{\nabla}\times\vec{E}(r) = 0$ and then use of Stoke's theorem to convert the surface integral into a contour integral: $\oint_S \vec{\nabla}\times\vec{E}(\vec{r})\cdot d\vec{A} = \oint_C \vec{E}(\vec{r})\cdot d\vec{\ell} = 0$
(closed surface) (closed contour)

Please see/read P435 Lecture Notes 3, p. 19.

8.) How is the boundary condition $\left.\frac{\partial V_{above}}{\partial n}\right|_{interface} - \left.\frac{\partial V_{below}}{\partial n}\right|_{interface} = -\left(\frac{\sigma}{\varepsilon_0}\right)$ obtained?
What is the physical meaning of this boundary condition?

Please see/read P435 Lecture Notes 3, p. 19.