Questions for P435 Lecture Notes 2

1.) What is the physical meaning of electric flux $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{E}(\vec{r}) \cdot \hat{n} dA}$.

In general, a flux of a physical quantity has SI units of (SI units of that quantity)/m^2.

2.) Think about the various aspects/details associated with the mathematical description of the process of going from a superposition of a very large number of discrete electrical charges contained in a well-defined/finite volume of space, to a *continuum* description, which is described by a volume electric charge density, $\rho(\vec{r})$ (SI units of Coulombs/m^3).

3.) What is/understand the physical meaning of the gradient of a scalar point function:

$$\vec{\nabla} f(\vec{r}) = \frac{\partial f(x,y,z)}{\partial x}\hat{x} + \frac{\partial f(x,y,z)}{\partial y}\hat{y} + \frac{\partial f(x,y,z)}{\partial z}\hat{z}$$ (in Cartesian/rectangular coordinates)

4.) What is/understand the physical meaning of the divergence of a vector point function:

$$\vec{\nabla} \cdot \vec{F}(\vec{r}) = \frac{\partial F_x(x,y,z)}{\partial x} + \frac{\partial F_y(x,y,z)}{\partial y} + \frac{\partial F_z(x,y,z)}{\partial z}$$ (in Cartesian/rectangular coordinates)

{See also P435 Lect. Notes 3}

5.) Know/understand the physical meaning of each/every mathematical symbol in Gauss's law

- in differential form: $\boxed{\vec{\nabla} \cdot \vec{E}(\vec{r}) = \rho(\vec{r})/\varepsilon_o}$

The divergence of $\vec{E}(\vec{r})$ at the (source!) point $\vec{r}$ is equal to the (total) volume electric charge density $\rho(\vec{r})$ at the same (source!) point $\vec{r}$, divided by the electric permittivity of free space ε_o.

- and also in integral form: $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{E}(\vec{r}) \cdot \hat{n} dA = Q_{encl}/\varepsilon_o}$

The (total) electric flux passing through a closed surface *S* is equal to the (total) electric charge enclosed by the surface S divided by the electric permittivity of free space ε_o.

6.) Know/understand how to use Gauss's law – see e.g. applications of, in P435 Lect. Notes. 2

7.) What is/understand the physical meaning of the curl of a vector point function:

$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$ (in Cartesian/rectangular coordinates)

{See also P435 Lect. Notes 3}

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Answers to Questions for P435 Lecture Notes 2

1.) What is the physical meaning of electric flux $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{E}(\vec{r}) \cdot \hat{n} dA}$.

In general, a flux of a physical quantity has SI units of (SI units of that quantity)/m^2.

Electric flux $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{E}(\vec{r}) \cdot \hat{n} dA}$ is physically defined in an analogous way e.g. to that of magnetic flux $\boxed{\Phi_m \equiv \oint_S \vec{B}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{B}(\vec{r}) \cdot \hat{n} dA}$, in either case, physically it can be thought of as the number of electric (or magnetic) field lines per unit area passing through (here) a closed surface *S*. However, we also know that for the electric field case, from Gauss' law $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{E}(\vec{r}) \cdot \hat{n} dA = Q_{encl}/\varepsilon_o}$ which is non-zero if electric charge is present within the enclosing surface *S*, whereas for the magnetic field case, we will see that for a closed surface *S* that $\boxed{\Phi_m \equiv \oint_S \vec{B}(\vec{r}) \cdot d\vec{A} = \oint_S \vec{B}(\vec{r}) \cdot \hat{n} dA \equiv 0}$ because magnetic charges do not exist in nature {if they do exist, they are exceedingly rare – none have ever been observed, despite many people searching for the existence of so-called magnetic monopoles, including yours truly…}.

Note that these relations *also* tell us that electric field lines originate from (terminate on) positive (negative) electric charges, respectively, whereas magnetic field lines close on themselves, since there are no magnetic charges!

2.) Think about the various aspects/details associated with the mathematical description of the process of going from a superposition of a very large number of discrete electrical charges contained in a well-defined/finite volume of space, to a *continuum* description, which is described by a volume electric charge density, $\rho(\vec{r})$ (SI units of Coulombs/m^3).

Aside from the microscopic => macroscopic averaging process associated with the physics over an infinitesimal volume element $d\tau$, this also entails the formal/rigorous mathematical procedure associated with this process.

3.) What is/understand the physical meaning of the gradient of a scalar point function:

$$\vec{\nabla} f(\vec{r}) = \frac{\partial f(x,y,z)}{\partial x}\hat{x} + \frac{\partial f(x,y,z)}{\partial y}\hat{y} + \frac{\partial f(x,y,z)}{\partial z}\hat{z} \quad \text{(in Cartesian/rectangular coordinates)}$$

In one dimension, e.g. $\frac{\partial f(x,y,z)}{\partial x}$ physically is the *x*-slope of the function $f(\vec{r}) = f(x,y,z)$, thus $\vec{\nabla} f(\vec{r}) = \frac{\partial f(x,y,z)}{\partial x}\hat{x} + \frac{\partial f(x,y,z)}{\partial y}\hat{y} + \frac{\partial f(x,y,z)}{\partial z}\hat{z}$ is the 3-D vector slope of the scalar point function $f(\vec{r}) = f(x,y,z)$ at the point $\vec{r} = (x,y,z)$.

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4.) What is/understand the physical meaning of the <u>divergence</u> of a vector point function:

$$\vec{\nabla}\bullet\vec{F}(\vec{r}) = \frac{\partial F_x(x,y,z)}{\partial x} + \frac{\partial F_y(x,y,z)}{\partial y} + \frac{\partial F_z(x,y,z)}{\partial z}$$ (in Cartesian/rectangular coordinates)

{See also P435 Lect. Notes 3} Check out the pix on the first page of P435 Lecture notes 3 to get a better physical understanding of divergence & curl of a vector point function.

5.) Know/understand the physical meaning of each/every mathematical symbol in Gauss's law

- in <u>differential</u> form: $\boxed{\vec{\nabla}\bullet\vec{E}(\vec{r}) = \rho(\vec{r})/\varepsilon_o}$

The divergence of $\vec{E}(\vec{r})$ at the (source!) point $\vec{r}$ is equal to the (total) volume electric charge density $\rho(\vec{r})$ at the same (source!) point $\vec{r}$, divided by the electric permittivity of free space ε_o.

- and also in <u>integral</u> form: $\boxed{\Phi_E \equiv \oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \oint_S \vec{E}(\vec{r})\bullet\hat{n}dA = Q_{encl}/\varepsilon_o}$

The (total) electric flux passing through a closed surface *S* is equal to the (total) electric charge enclosed by the surface S divided by the electric permittivity of free space ε_o.

6.) Know/understand how to <u>use</u> Gauss's law – please see/explicitly work through the details yourselves the applications of Gauss' law in P435 Lect. Notes. 2

7.) What is/understand the physical meaning of the <u>curl</u> of a vector point function:

$$\vec{\nabla}\times\vec{F}(\vec{r}) = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$ (in Cartesian/rectangular coordinates)

{See also P435 Lect. Notes 3} Check out the pix on the first page of P435 Lecture notes 3 to get a better physical understanding of divergence & curl of a vector point function.

In the late 1800's/early 1900's, the notation used for $\vec{\nabla}\times\vec{F}(\vec{r})$ back then, in particular, in Europe was $rot\,\vec{F}(\vec{r})$ for "rotor of *F*", which is fine e.g. for circulation/vorticity associated with whirlpools in fluid flow – i.e. the circulation per unit area of a vector field $\vec{F}(\vec{r})$, mathematically defined as: $\vec{\nabla}\times\vec{F}(\vec{r}) = \lim_{S\to 0}\frac{\oint_C \vec{F}(\vec{r})\bullet d\vec{\ell}}{S}$ However, what about ***<u>shear</u>*** fields, which also have non-zero $\vec{\nabla}\times\vec{F}(\vec{r})$? Thus "curl of *F*" is used today.

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