Physics 401

Pulses in Transmission Lines

(week of February 14th)

- Comment on the Lab-Quiz
- Pulses in Transmission Lines

Quiz → Lab-Discussion

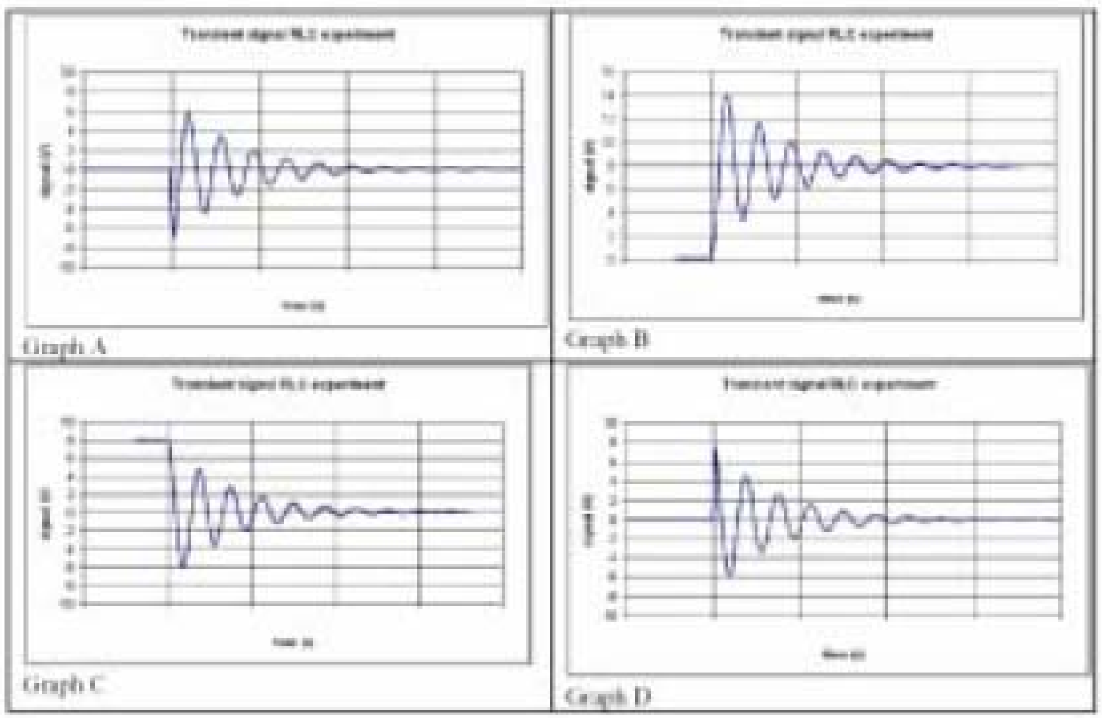
- Don’t work on it at home!

- TAs will distribute the lab-discussion sheets in class.

  → collaborate with other students + TA in answering the questions!

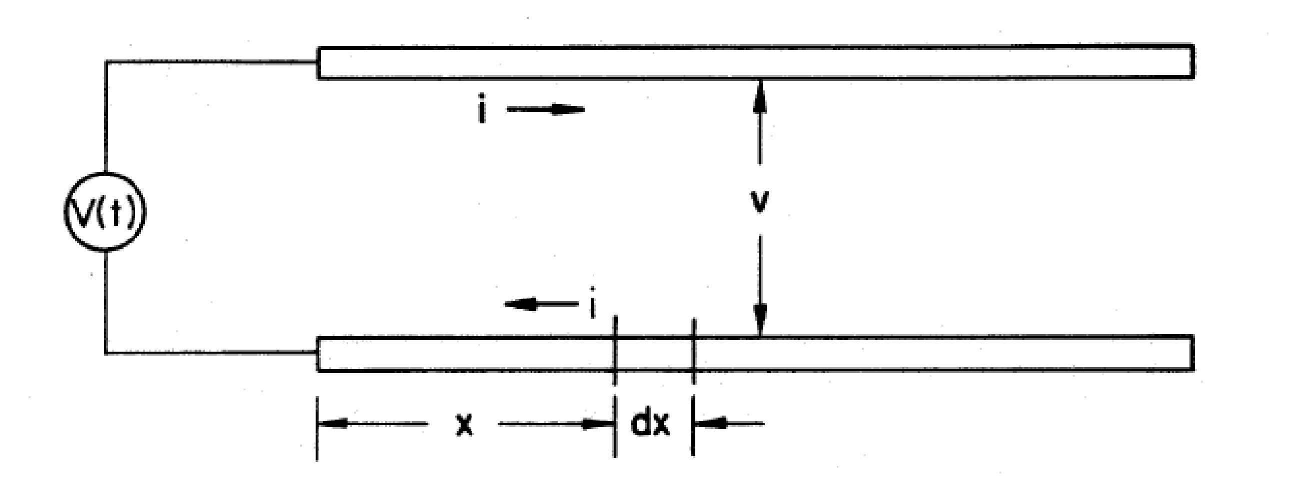
Comments:

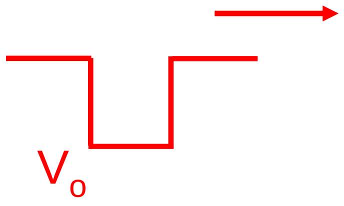
- RLC pre-lab



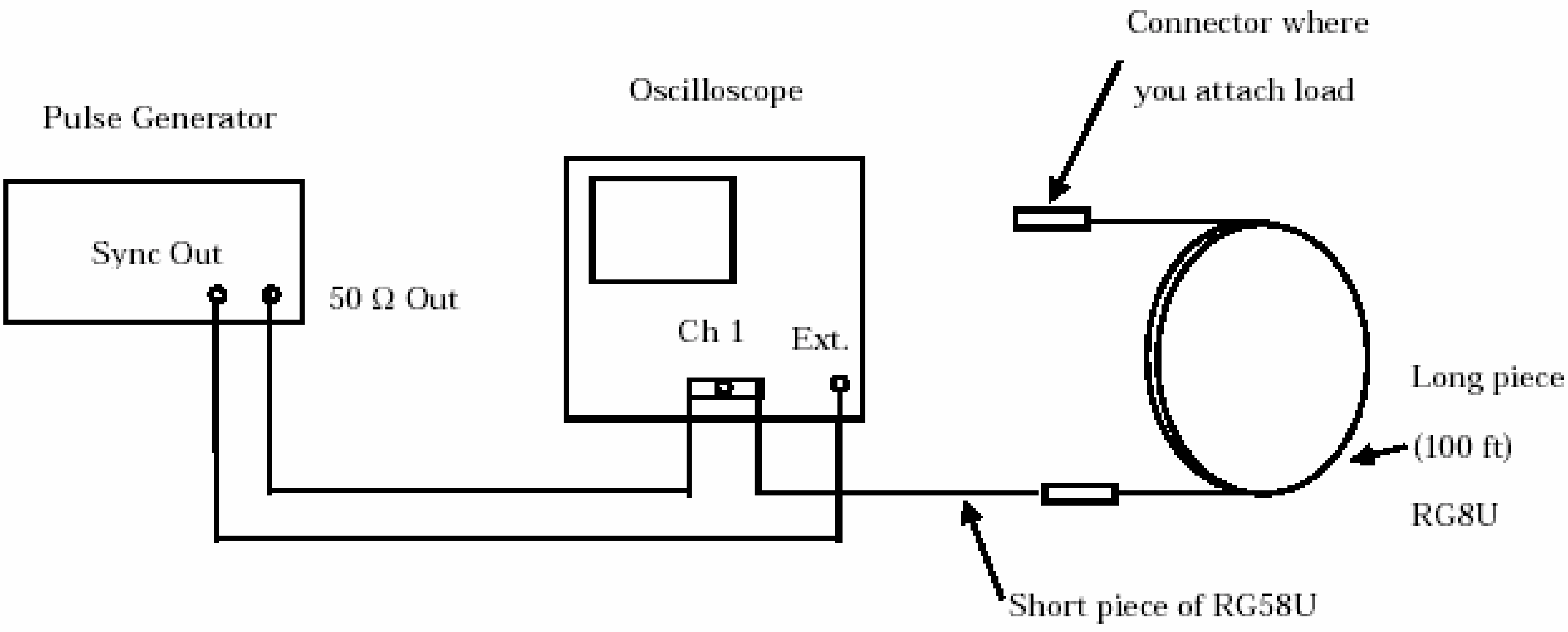
- Counting Experiment: How to treat the error on the background.

Pulses in Transmission Lines

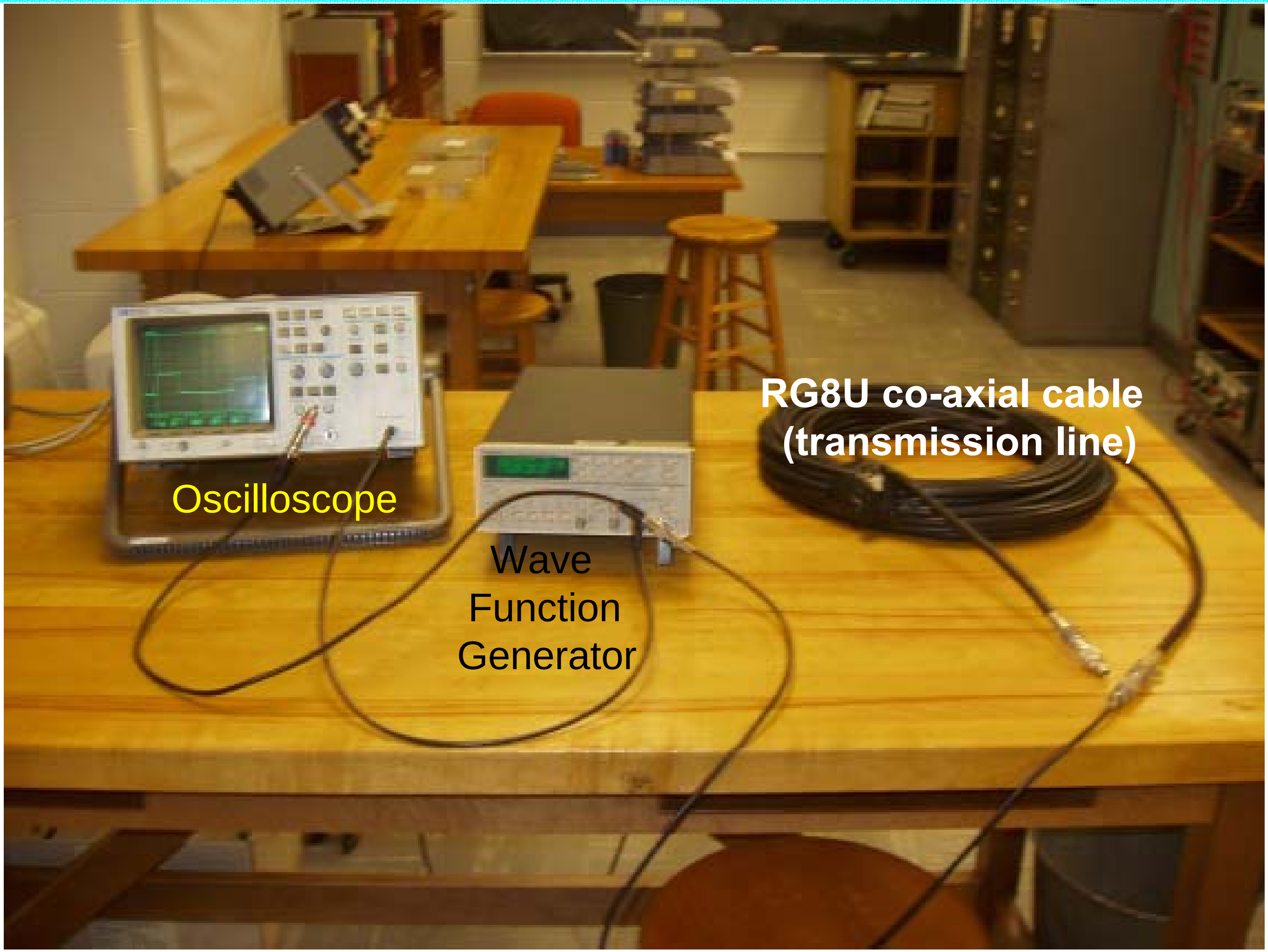




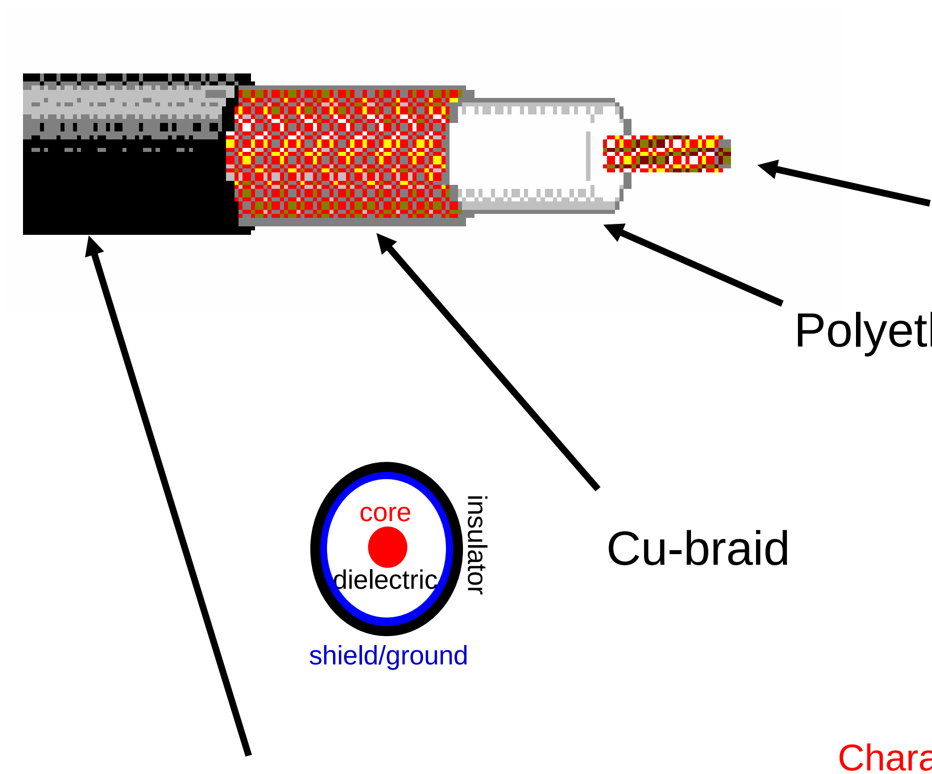
Experimental Setup I

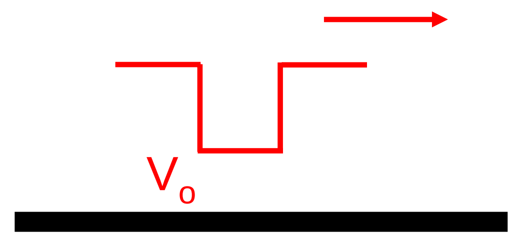


Experimental Setup II



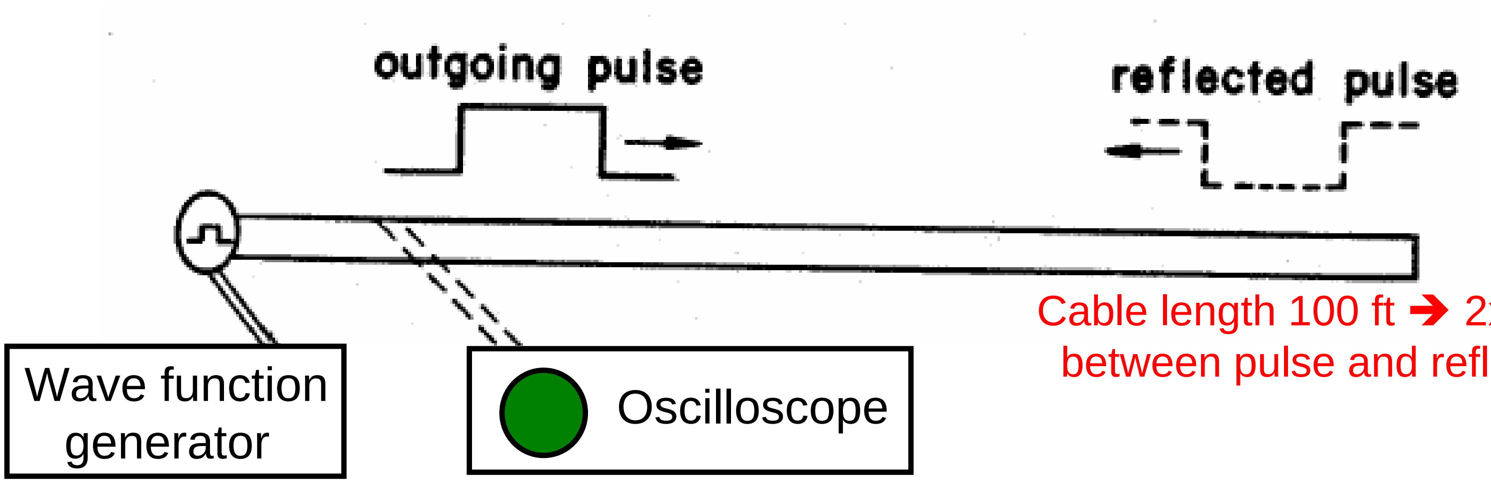
Co-axial Cables (eg. RG8U)





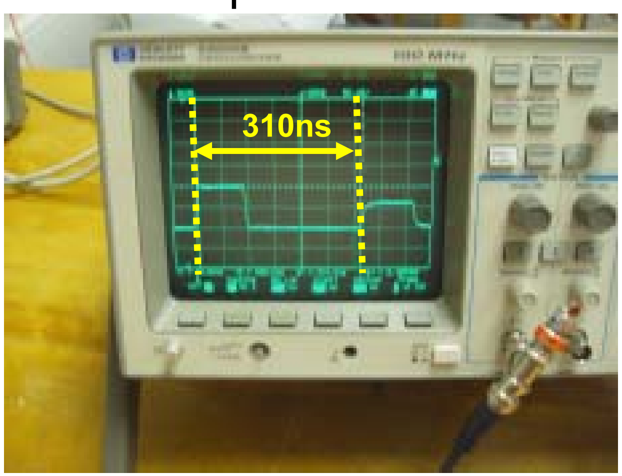
Characteristic Impedance	52.0	Ohms
Inductance	0.079	µH/ft
Capacitance Conductor to Shield	28.5	pF/ft
Velocity of Propagation	0.66	c
Delay	1.54	ns/ft

Signal Propagation

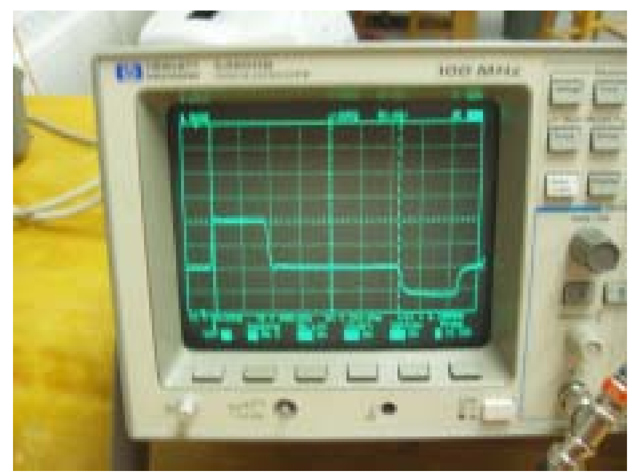


Cable length 100 ft ➔ 2x154ns between pulse and reflection

open line



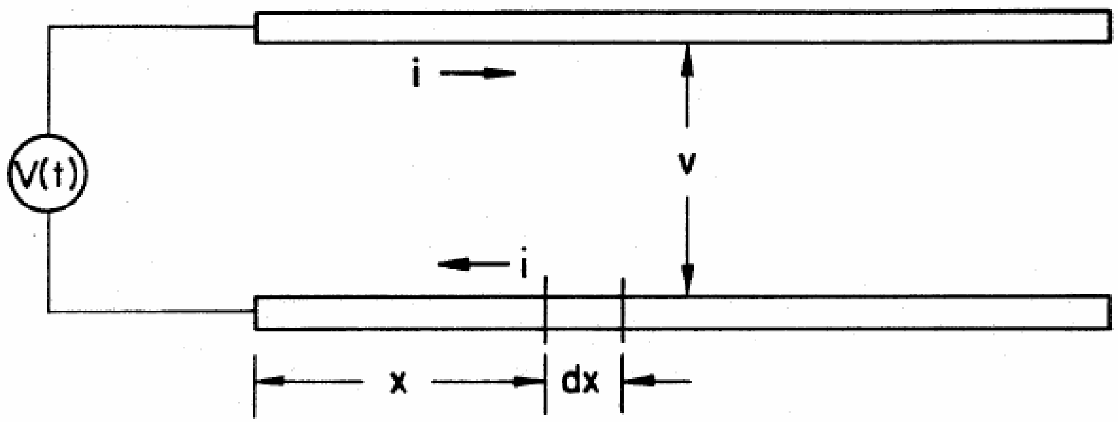
shorted line



The Wave Equation

$$di = -(Cdx)\frac{dV}{dt},$$

C = capacitance per unit length



$$dV = -(Ldx)\frac{di}{dt},$$

L = inductance per unit length

$$\frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{\partial t}$$

The Wave Equation

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{\partial t} \qquad \frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$\frac{\partial}{\partial x}$ $\qquad$ $\frac{\partial}{\partial t}$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \qquad \frac{\partial^2 i}{\partial t \partial x} = -C\frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad \frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2}$$

Voltage and Current Waves

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad \frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2}$$

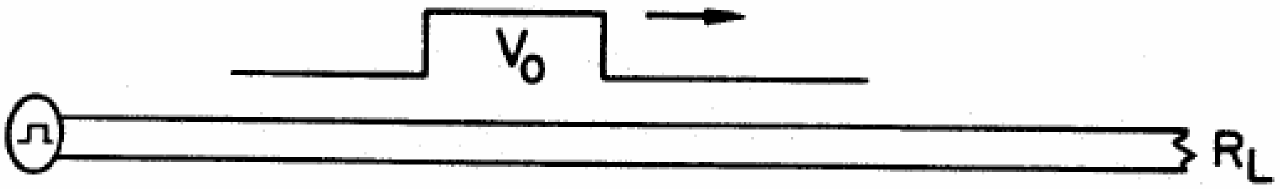
$$V(x,t) = V_0 \sin\omega(t - \frac{x}{v}), \quad v = \frac{1}{\sqrt{LC}}$$

$$i(x,t) = i_0 \sin\omega(t - \frac{x}{v}), \quad v = \frac{1}{\sqrt{LC}}$$

$$V(x,t) = \sqrt{\frac{L}{C}}i(x,t) = Z_k i(x,t)$$

Z_k: characteristic Impedance

Reflection of Pulses at Resistive Loads



$$R_L = \frac{V}{i} = \frac{V_r + V_i}{V_i - V_r} Z_k$$

$$\Rightarrow \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

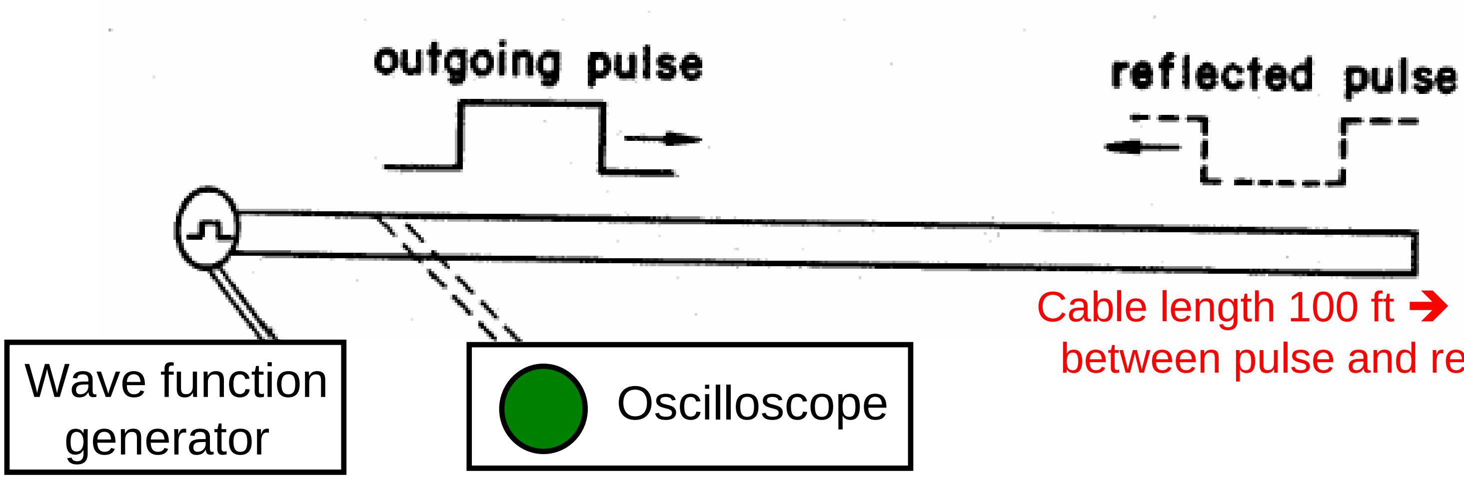
$$\Rightarrow \quad V_r = V_i = V_0 \quad , \quad \text{if} \ \ R_L = \infty$$

Termination resistor disconnected

$$\Rightarrow \quad V_r = -V_i = -V_0 \quad , \quad \text{if} \ \ R_L = 0$$

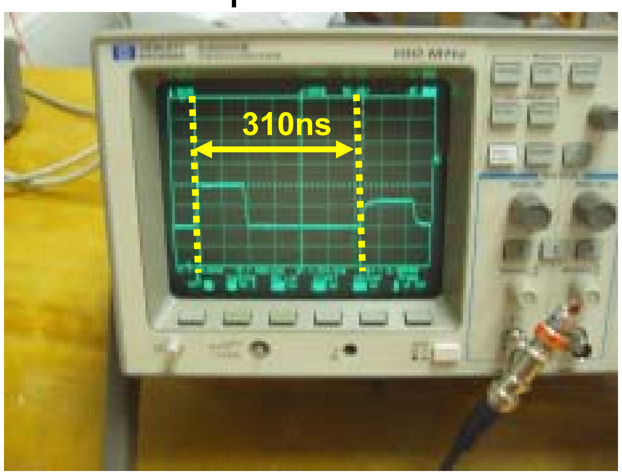
Termination resistor shorted

Incoming vs Reflected Wave

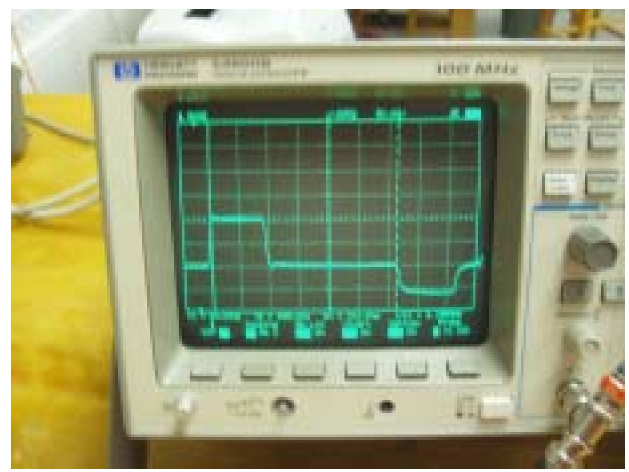


Cable length 100 ft ➔ 2x154ns between pulse and reflection

open line



shorted line



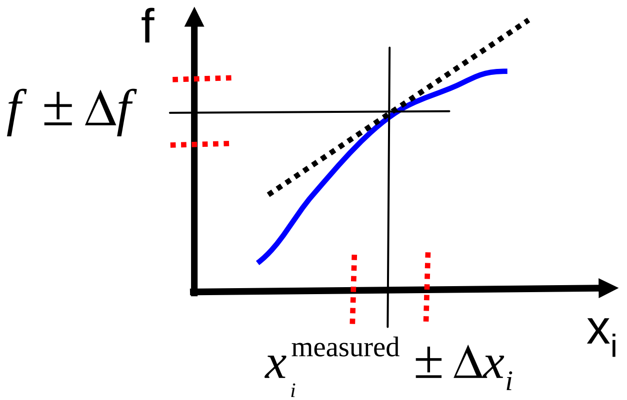
Error Propagation

Derive f from measured quantities quantities x_i+/-Δx_i

$$f = f(x_1, \ldots, x_n)$$

Error Δf on f:

$$\Delta f(x_i, \Delta x_i) = \sqrt{\sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i} \right]^2 \cdot \Delta x_i^2}$$



Example: Frequency in LC circuit

Derive frequency *f* from measured inductance *L*+/-Δ*L* and capacitance *C*+/-Δ*C*

$$f(L,C) = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 503.6 \pm 56.3\text{Hz},$$

$$L = 10 \pm 1\text{mH},\ \ C = 10 \pm 2\mu\text{F}$$

Error Δf on f:

$$\Delta f(L,C,\Delta L,\Delta C) = \sqrt{\left[\frac{\partial f}{\partial L}\right]^2 \cdot \Delta L^2 + \left[\frac{\partial f}{\partial C}\right]^2 \Delta C^2}$$

$$\frac{\partial f}{\partial L} = \frac{-1}{4\pi} C^{-\frac{1}{2}} L^{-\frac{3}{2}}$$

$$\frac{\partial f}{\partial C} = \frac{-1}{4\pi} L^{-\frac{1}{2}} C^{-\frac{3}{2}}$$