Exercises on Bound States in One Dimension

1. We argued that both the wave function and its derivative is continuous across a boundary between two finite potential steps. Is the double derivative continuous as well? Give an argument to defend your answer.

2. An electron is bound in a stepwise constant potential well where $V = \infty$ outside of a region from $0 < x < 0.15\ nm$. The potential energy in the region from $0 < x < 0.10\ nm$ is $V = 2\ eV$ and the electron has a wavelength of $\lambda = 0.2\ nm$. In the region from $0.1 < x < 0.15\ nm$, the potential is unknown and the electron has a wavelength of $\lambda = 0.10\ nm$.

   (a) Is the electron in the ground, first excited, or second excited state?

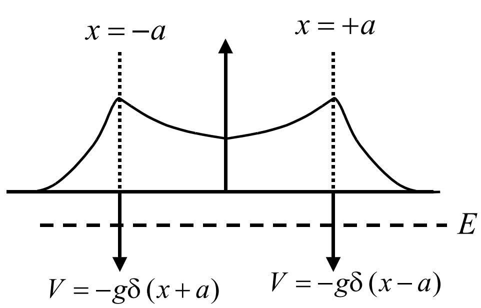
   (b) What is the total energy of the electron?

   (c) What is the potential in the region $0.1 < x < 0.15\ nm$?

   (d) Apart from any overall normalization constant, write a fully explicit expression for wave function in the two regions.

   (e) Find the probability that the electron can be found in the region $0.1 < x < 0.15\ nm$?

3. An electron is bound in the double δ- function well shown below. I also crudely sketched its wave function.



   (a) Assume the electron has an energy of -E when in the state pictured above. Obtain an expression for the strength g of the δ-function in terms of a,m and physical constants as needed.

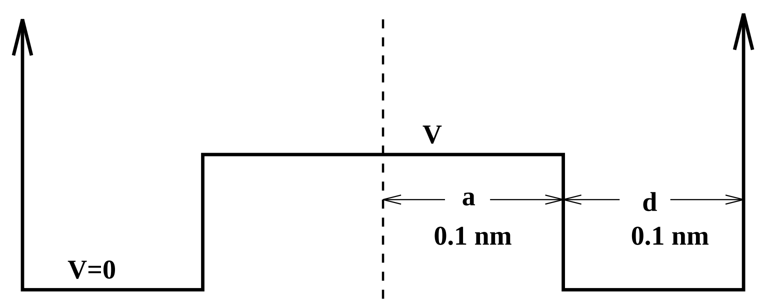
(b) Check your answer by going to the limit $a \to 0$ where the problem degenerates into a single δ-function of strength $-2g$.

(c) Find a numerical value for g for the case of an electron bound with an energy of $E = -10\ eV$ in a well with $a = 0.2\ nm$.

4. Obtain the transcendental equation for the energies of the odd parity solutions of the symmetric finite square of depth V and width a. Graphically find all odd parity bound state solutions for the case of an electron bound in a $V = 50\ eV$ deep well which is $a = 0.2\ nm$ wide.

5. Find the minimum width a required such that a finite square well of depth $V = 50\ eV$ has 4 bound states.

6. An electron is placed in the double well depicted below with $a = d = 0.1\ nm$



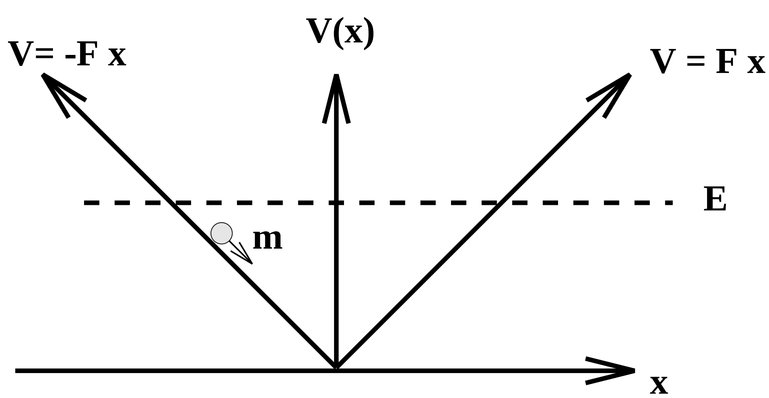
(a) Obtain a transcendental equation which can be solved graphically for the even parity solutions with $E > V$.

(b) Solve for the lowest energy even solution with $E > V$ for a well with $V = 5\ eV$.

(c) Solve for the lowest energy even solution with $E > V$ for a well with $V = 10\ eV$. Is this lowest energy, $E > V$, even solution the ground state? Hint: For $d = 0.1\ nm$ what is the minimum value of V such that the ground state is of the form $\psi \propto \cosh(\beta x)$ in the hump region. Draw the wave function for a case where the ground state energy equals to V.

7. It the atom were modeled as a harmonic oscillator, a characteristic size corresponding to the dimensionless $\xi = 1$ might be $x = 0.2\ nm$. How would the frequency of such an oscillator compare to the frequency of visible light $400\ nm < \lambda < 700\ nm$. Assume that the electron is the mass that oscillates.

8. Make a qualitative sketch for the wave function for the fifth excited state of a particle in the double ramp potential show below:



9. Demonstrate that a_- acts as a lowering operator for the harmonic oscillator stationary states. Feel free to assume $H = a_+a_- + \hbar\omega/2$ and $[a_+, a_-] = -\hbar\omega$.

10. In this problem , I ask you to compute $< p^2 >$ and $< x^2 >$ for the ψ_n of the harmonic oscillator by writing the $\check{x}$ and $\check{p}$ operators in terms of the raising and lowering operators $a_\pm$. The basic forms are $\check{p} \propto a_+ + a_-$ and $\check{x} \propto a_+ - a_-$ but you need to fill in the missing details. Assume without proof that the stationary states are orthonormal $< \psi_n|\psi_m >= \delta_{mn}$ and the facts in developed in lecture concerning the Hamiltonian and the properties of these operators.

    (a) Show $< x >=< p >= 0$ for all stationary states ψ_n.

    (b) Compute $< \psi_n|x^2|\psi_n >$ as a function of n, m, ω and physical constants.

    (c) Check your answer to part (b) by direct calculation for the case $\psi_0 \propto \exp(-\xi^2/2)$

(d) Compute $< \psi_n|p^2|\psi_n >$ as a function of n, m, ω and physical constants and evaluate the "uncertainty product" $\sigma_x\ \sigma_p$ for ψ_n.

(e) Show that $< T >=< V >$ for the general stationary state ψ_n where T is kinetic energy and V is potential.

11. Assume the ground state of the harmonic oscillator is $\psi_o \propto \exp(-\xi^2/2)$. Feel free to normalize in ξ.

    (a) Properly normalize it.

    (b) Construct ψ_1 using the normalization preserving ladder operator.

    (c) Verify the your ψ_1 is properly normalized.

Useful Integals

$\int_0^\infty\ dy\ exp(-y^2)$	$\int_0^\infty\ dy\ y\ exp(-y^2)$	$\int_0^\infty\ dy\ y^2\ exp(-y^2)$
$\sqrt{\pi}/2$	$1/2$	$\sqrt{\pi}/4$