

Physics 325

Classical Mechanics I

George Gollin

University of Illinois at Urbana-Champaign

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Classical Mechanics I

Physics 325

Welcome!

Here is an introduction and syllabus for the course.

Take note:

- We'll use calculus all the time.
- Attendance at lectures is mandatory. I will penalize you for unexcused absences.
- Come to the Monday night problem sessions.
- No calculators or computers are to be used as computational aids or as portals to access reference sources unless I explicitly give the OK for a particular problem. You are not to use them when working problem sets, problem session exercises, or exams.
- No laptop, cellphone, iPhone, iPad usage during class, please. I do not object to snacks and beverages.

- I encourage you to work together on problem sets, but please make sure everyone in your group participates in generating the solutions and understands how they were obtained. You are not to submit solutions that you have found on the web, in books of worked problems, in archival copies of earlier offerings of the course, or any other source of that nature. That would be cheating: submitting work for a grade that is not your own.
- There will be two (evening) midterms, no time limit. one final exam.
- Homework counts a lot. Late homework will be penalized 50%. Homework more than one week late will not be graded.

Text: *Classical Dynamics of Particles and Systems, 5th edition*, Stephen T. Thornton, Jerry B. Marion. Note: the fourth edition is fine too, and MUCH less expensive. The third edition has a chapter on chaos and nonlinear dynamics, and you can always borrow a later edition when it's time to take a look at that material next semester if you've found an even earlier edition. (The fourth and third editions can be purchased through online merchants.)

My lecture notes will be available at the Illini Union Bookstore. You should buy your own copy and bring it to class.

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Physics 325 course calendar

Week	Day	Date	Time	Room	Class activity	Topic	Due
1	Mon	8/22	8:30 am	LLP 144	Lecture 1	Newtonian Mechanics in one dimension	
1	Mon	8/22			No problem sessions		
1	Tue	8/23			No problem sessions		
1	Wed	8/24	8:30 am	LLP 144	Lecture 2	Newtonian Mechanics in one dimension	
2	Mon	8/29	8:30 am	LLP 144	Lecture 3	Newtonian Mechanics in one dimension	
2	Mon	8/29	6:00 pm	LLP 236	Problem session 1		
2	Mon	8/29	7:00 pm	LLP 236	Problem session 1		
2	Mon	8/29	8:00 pm	LLP 236	Problem session 1		
2	Tue	8/30	7:00 pm	LLP 236	Problem session 1		
2	Tue	8/30	8:00 pm	LLP 236	Problem session 1		
2	Wed	8/31	8:30 am	LLP 144	Lecture 4	Dynamics in three dimensions	HW 1
3	Mon	9/5			Labor Day		
3	Tue	9/6			No problem sessions		
3	Wed	9/7	8:30 am	LLP 144	Lecture 5	Dynamics in three dimensions	HW 2
4	Mon	9/12	8:30 am	LLP 144	Lecture 6	Dynamics in three dimensions	
4	Mon	9/12	6:00 pm	LLP 236	Problem session 2		
4	Mon	9/12	7:00 pm	LLP 236	Problem session 2		
4	Mon	9/12	8:00 pm	LLP 236	Problem session 2		
4	Tue	9/13	7:00 pm	LLP 236	Problem session 2		
4	Tue	9/13	8:00 pm	LLP 236	Problem session 2		
4	Wed	9/14	8:30 am	LLP 144	Lecture 7	Systems of particles	HW 3
5	Mon	9/19			No lecture: GG away		
5	Mon	9/19	6:00 pm	LLP 236	Problem session 3		
5	Mon	9/19	7:00 pm	LLP 236	Problem session 3		
5	Mon	9/19	8:00 pm	LLP 236	Problem session 3		
5	Tue	9/20	7:00 pm	LLP 236	Problem session 3		
5	Tue	9/20	8:00 pm	LLP 236	Problem session 3		
5	Wed	9/21	8:30 am	LLP 144	Lecture 8	Oscillations	HW 4
6	Mon	9/26	8:30 am	LLP 144	Lecture 9	Oscillations	
6	Mon	9/26	6:00 pm	LLP 236	Problem session 4		
6	Mon	9/26	7:00 pm	LLP 236	Problem session 4		
6	Mon	9/26	8:00 pm	LLP 236	Problem session 4		
6	Tue	9/27	7:00 pm	LLP 236	Problem session 4		
6	Tue	9/27	8:00 pm	LLP 236	Problem session 4		
6	Wed	9/28	8:30 am	LLP 144	Lecture 10	Oscillations	HW 5
7	Mon	10/3	8:30 am	LLP 144	No lecture: GG away		
7	Mon	10/3	6:00 pm	LLP 236	Problem session 5		
7	Mon	10/3	7:00 pm	LLP 236	Problem session 5		
7	Mon	10/3	8:00 pm	LLP 236	Problem session 5		
7	Tue	10/4	7:00 pm	LLP 236	Problem session 5		
7	Tue	10/4	8:00 pm	LLP 236	Problem session 5		
7	Wed	10/5	8:30 am	LLP 144	Lecture 11	Driven oscillations	HW 6
7	Thu	10/6	7:00 pm	TBA	Exam 1	Material from weeks 1 - 6	

8	Mon	10/10	8:30 am	LLP 144	Lecture 12	Driven oscillations	
8	Mon	10/10	6:00 pm	LLP 236	Problem session 6		
8	Mon	10/10	7:00 pm	LLP 236	Problem session 6		
8	Mon	10/10	8:00 pm	LLP 236	Problem session 6		
8	Tue	10/11	7:00 pm	LLP 236	Problem session 6		
8	Tue	10/11	8:00 pm	LLP 236	Problem session 6		
8	Wed	10/12	8:30 am	LLP 144	Lecture 13	Driven oscillations	HW 7
9	Mon	10/17	8:30 am	LLP 144	Lecture 14	Impulse response	
9	Mon	10/17	6:00 pm	LLP 236	Problem session 7		
9	Mon	10/17	7:00 pm	LLP 236	Problem session 7		
9	Mon	10/17	8:00 pm	LLP 236	Problem session 7		
9	Tue	10/18	7:00 pm	LLP 236	Problem session 7		
9	Tue	10/18	8:00 pm	LLP 236	Problem session 7		
9	Wed	10/19	8:30 am	LLP 144	Lecture 15	Impulse response	HW 8
10	Mon	10/24	8:30 am	LLP 144	Lecture 16	Impulse response	
10	Mon	10/24	6:00 pm	LLP 236	Problem session 8		
10	Mon	10/24	7:00 pm	LLP 236	Problem session 8		
10	Mon	10/24	8:00 pm	LLP 236	Problem session 8		
10	Tue	10/25	7:00 pm	LLP 236	Problem session 8		
10	Tue	10/25	8:00 pm	LLP 236	Problem session 8		
10	Wed	10/26	8:30 am	LLP 144	Lecture 17	Motion in rotating frames	HW 9
11	Mon	10/31	8:30 am	LLP 144	Lecture 18	Motion in rotating frames	
11	Mon	10/31	6:00 pm	LLP 236	Problem session 9		
11	Mon	10/31	7:00 pm	LLP 236	Problem session 9		
11	Mon	10/31	8:00 pm	LLP 236	Problem session 9		
11	Tue	11/1	7:00 pm	LLP 236	Problem session 9		
11	Tue	11/1	8:00 pm	LLP 236	Problem session 9		
11	Wed	11/2	8:30 am	LLP 144	Lecture 19	Fluid dynamics	HW 10
12	Mon	11/7	8:30 am	LLP 144	Lecture 20	Fluid dynamics	
12	Mon	11/7	6:00 pm	LLP 236	Problem session 10		
12	Mon	11/7	7:00 pm	LLP 236	Problem session 10		
12	Mon	11/7	8:00 pm	LLP 236	Problem session 10		
12	Tue	11/8	7:00 pm	LLP 236	Problem session 10		
12	Tue	11/8	8:00 pm	LLP 236	Problem session 10		
12	Wed	11/9	8:30 am	LLP 144	Lecture 21	Fluid dynamics	HW 11
13	Mon	11/14	8:30 am	LLP 144	Lecture 22	Lagrangians and calculus of variations	
13	Mon	11/14	6:00 pm	LLP 236	Problem session 11		
13	Mon	11/14	7:00 pm	LLP 236	Problem session 11		
13	Mon	11/14	8:00 pm	LLP 236	Problem session 11		
13	Tue	11/15	7:00 pm	LLP 236	Problem session 11		
13	Tue	11/15	8:00 pm	LLP 236	Problem session 11		
13	Wed	11/16	8:30 am	LLP 144	Lecture 23	Lagrangians and calculus of variations	HW 12
13	Thu	11/17	7:00 PM	TBA	Exam 2	Material from weeks 6 - 12	

	Mon	11/21			Thanksgiving break		
	Tue	11/22			Thanksgiving break		
	Wed	11/23			Thanksgiving break		
14	Mon	11/28	8:30 am	LLP 144	Lecture 24	Lagrangians and calculus of variations	
14	Mon	11/28	6:00 pm	LLP 236	Problem session 12		
14	Mon	11/28	7:00 pm	LLP 236	Problem session 12		
14	Mon	11/28	8:00 pm	LLP 236	Problem session 12		
14	Tue	11/29	7:00 pm	LLP 236	Problem session 12		
14	Tue	11/29	8:00 pm	LLP 236	Problem session 12		
14	Wed	11/30	8:30 am	LLP 144	Lecture 25	Lagrangians and calculus of variations	HW 13
15	Mon	12/5	8:30 am	LLP 144	Lecture 26	Lagrangians and calculus of variations	
15	Mon	12/5	6:00 pm	LLP 236	Problem session 13		
15	Mon	12/5	7:00 pm	LLP 236	Problem session 13		
15	Mon	12/5	8:00 pm	LLP 236	Problem session 13		
15	Tue	12/6	7:00 pm	LLP 236	Problem session 13		
15	Tue	12/6	8:00 pm	LLP 236	Problem session 13		
15	Wed	12/7	8:30 am	LLP 144	Lecture 27	Lagrangians and calculus of variations	HW 14
	Fri	12/16	8:00 AM	TBA	Final exam	Entire course	

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