The Plan

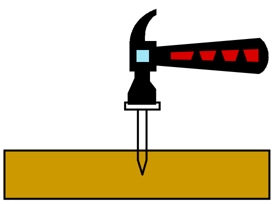
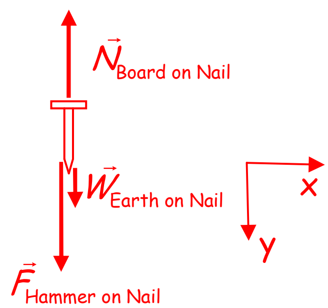
This week you'll practice applying Newton's Second Law (a.k.a. $\vec{F}_{net} = m\vec{a}$). In order to use this relationship, you must first be able to identify the forces on an object and their directions. This is accomplished by drawing a free body diagram (i.e. drawing the object you're interested in, the forces on it, and NOTHING ELSE). After you have a free body diagram, you can use Newton's 2^{nd} Law to write an equation for the acceleration components of your object in each direction of interest (i.e. one equation each for x- and y-directions). You can then solve these equations simultaneously for unknown forces or accelerations. To practice these skills, today's activities are divided into two groups:

1) In DQ1, your group will generate free body diagrams (FBDs) and $\vec{F}_{net} = m\vec{a}$ equations in two dimensions. The goal is to practice identifying forces and producing equations from them.

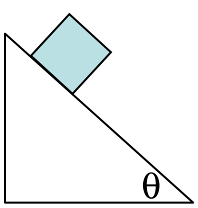
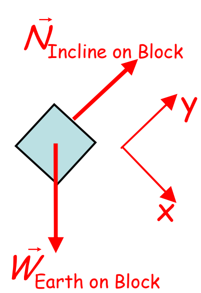
2) In the rest of the questions, your group will generate or use FBDs to solve for unknown forces or accelerations (or both). These exercises build on the skills you practice in DQ1.

DQ1) Talk to the other people at your table in order to generate a free body diagram for the requested object in each of the following situations. After you have discussed the diagrams with each other, generate equations expressing Newton's Second Law. (Note: you do not have to solve these equations, simply set them up) When you generate symbols to represent different forces, make sure you include both the object generating the force and the object on which the force acts.

a) A nail is hit with a hammer, accelerating into a board of wood. Draw the free body diagram for the nail. Generate Newton's Second Law equations for the nail.

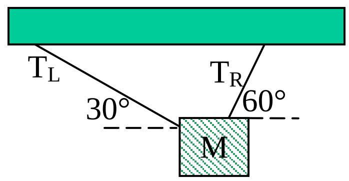
Situation	Nail's Free Body Diagram	Newton's 2nd Law Eqns. For Nail
(Nail hit with a hammer) 		y-direction $F_{net,y} = ma_y$ $F_{HN} + W_{EN} - N_{BN} = m_{Nail}a_y$

b) A block slides down a frictionless incline. Draw the free body diagram for the block. Generate Newton's Second Law equations for the block in each of two perpendicular directions.

Situation	Block's Free Body Diagram	Newton's 2nd Law Eqns. For Block
(Block on an Incline) 		x-direction $F_{net,x} = ma_x$ $W\sin\theta = ma_x$ y-direction $F_{net,y} = ma_y$ $N - W\cos\theta = ma_y$

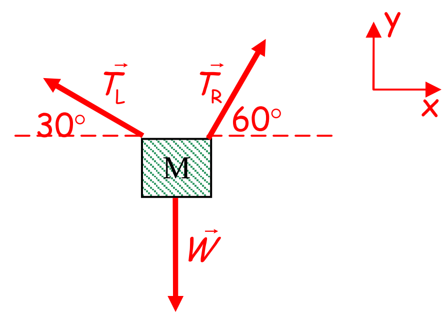
Here we chose our x-axis to coincide with the expected direction of the acceleration.

DQ2) A sign of mass M is suspended from the ceiling. The string on the right is connected to the block so that it makes an angle of 60° with respect to the horizontal. The string on the left is connected to the block so that it makes an angle 30° with respect to the horizontal.



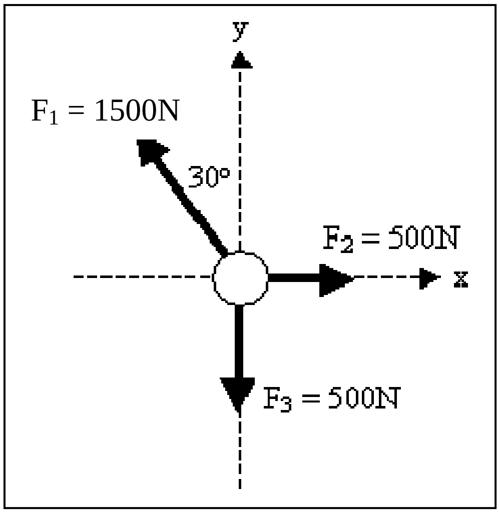
Talk to your group to help you draw a free body diagram for the block. Find the two tensions, T_L and T_R, in terms of M and g.

In this problem, we know the acceleration of the sign… it's zero! (the mass always remains at rest implies that its velocity never changes) We can use Newton's 2nd Law to relate this known acceleration to the unknown tensions.

Draw a free body diagram for the sign to identify all the forces on it. Since we're given the angles with respect to the horizontal, let's use axes in the horizontal and vertical directions (then we don't have to do any more geometry to find new angles).	
Write Newton's Second Law for the sign, breaking it into components in the x- and y-directions. We'll start with the x-direction. We know that the acceleration in the x-direction is zero. We're left with one equation and two unknowns (the two tension magnitudes). We can use the y-components of Newton's Second Law to get another equation.	$\vec{F}_{net} = M\vec{a}$ x-direction: $F_{net,x} = Ma_x$ $T_{Rx} + T_{Lx} + W_x = M(0)$ $\left(T_R \cos 60^\circ\right) + \left(-T_L \cos 30^\circ\right) + (0) = 0$ $T_R \frac{1}{2} - T_L \frac{\sqrt{3}}{2} = 0$ $T_R = \sqrt{3} T_L$

Now's we'll use Newton's Second Law in the y-direction. We know that the acceleration in the y-direction is also zero.	$F_{net,y} = Ma_y$ $T_{Ry} + T_{Ly} + W_y = M(0)$ $(T_R \sin 60^\circ) + (T_L \sin 30^\circ) + (-Mg) = 0$ $T_R \frac{\sqrt{3}}{2} + T_L \frac{1}{2} - Mg = 0$
Now we can substitute the relation from the x-direction into the relation from the y-direction.	$T_R \frac{\sqrt{3}}{2} + T_L \frac{1}{2} - Mg = 0$ $(\sqrt{3}T_L)\frac{\sqrt{3}}{2} + T_L \frac{1}{2} - Mg = 0$ $T_L \frac{3}{2} + T_L \frac{1}{2} = Mg$ $2T_L = Mg$ $\boxed{T_L = \frac{Mg}{2}}$
Now we can plug this relation back into one of the previous equations to get the tension in the right string. (the x-direction equation looks simpler, so we'll use that one)	$T_R = \sqrt{3}TL$ $= \sqrt{3}\left(\frac{Mg}{2}\right)$ $\boxed{T_R = \frac{\sqrt{3}}{2}Mg}$

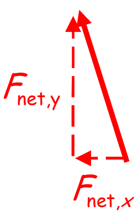
DQ3) Three forces act on an object as shown in the figure below. (F_2 points along the x direction and F_3 points along the -y direction). What is the magnitude of the net force acting on the object?



The net force on an object is the sum of all the forces on it. We simply need to add all the forces up, remembering to treat them as vectors. The vector relationship $\vec{F}_{net} = \sum_i \vec{F}_i$ actually stands for two scalar relationships:

$$F_{net,x} = \sum_i F_{i,x} = F_{1x} + F_{2x} + F_{3x}$$
$$= -(1500\text{ N})\sin 30^\circ + (500\text{ N}) + 0$$
$$= -250\text{ N}$$

$$F_{net,y} = \sum_i F_{i,y} = F_{1y} + F_{2y} + F_{3y}$$
$$= (1500\text{ N})\cos 30^\circ + 0 + (-500\text{ N})$$
$$= 799\text{ N}$$

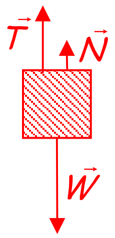


We can get the resultant net force magnitude using the Pythagorean Theorem.

$$F_{net} = \sqrt{(F_{net,x})^2 + (F_{net,y})^2} = \sqrt{(-250\ N)^2 + (799\text{ N})^2} = \boxed{837\ N}$$

DQ4) A rope is pulling a block of mass M across a slick surface with a constant tension T. Answer the following questions in terms of M, T, g, and θ (if needed).

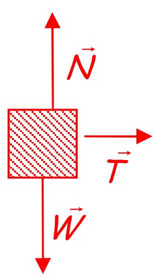
a) What is the normal force from the surface on the block if the rope is vertical? What is the acceleration of the block in this case? (assume that T is smaller than the weight of the object)



The block remains on the ground while the tension pulls upward. In this case, the acceleration of the block is zero (assuming the weight is greater than the tension). The normal force will be just enough to keep the block from crashing through the surface.

$$\vec{a} = 0$$
$$F_{net,y} = Ma_y = M(0)$$
$$T_y + N_y + W_y = 0$$
$$T + N + (-Mg) = 0$$
$$\boxed{N = Mg - T}$$

b) What is the normal force from the surface on the block if the rope is horizontal? What is the acceleration of the block in this case?



The block does not accelerate in the vertical direction (which we'll call the y-direction). We'll use Newton's Second Law in the x- and y-directions to find the Normal force and the acceleration.

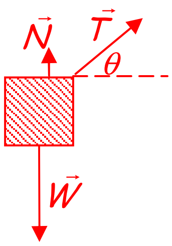
y-direction:

$$F_{net,y} = Ma_y = M(0)$$
$$N_y + T_y + W_y = 0$$
$$N + 0 + (-Mg) = 0$$
$$\boxed{N = Mg}$$

x-direction:

$$F_{net,x} = Ma_x$$
$$N_x + T_x + W_x = Ma_x$$
$$0 + T + 0 = Ma_x$$
$$\boxed{a_x = \frac{T}{M}}$$

c) What is the normal force from the surface on the block if the rope is at an angle θ with the horizontal? What is the acceleration of the block in this case? (assume that T is smaller than the weight of the object)



Again, we'll use Newton's Second Law, breaking the equation into an equation for each direction. We'll use vertical and horizontal as the y- and x-axes, respectively. Again, the block does not accelerate in the y-direction.

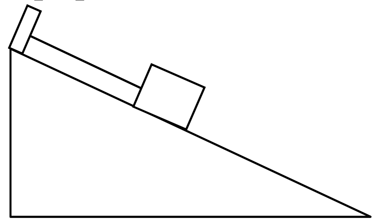
y-direction:

$$F_{net,y} = Ma_y = M(0)$$
$$N_y + T_y + W_y = 0$$
$$N + (T\sin\theta) + (-Mg) = 0$$
$$\boxed{N = Mg - T\sin\theta}$$

x-direction:

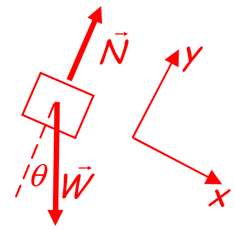
$$F_{net,x} = Ma_x$$
$$Nx + Tx + Wx = Ma_x$$
$$0 + (T\cos\theta) + 0 = Ma_x$$
$$\boxed{a_x = \frac{T}{M}\cos\theta}$$

DQ5) A block with mass M sits on a ramp inclined at an angle θ degrees above the horizontal, held in place with a rope parallel to the surface (See diagram below).



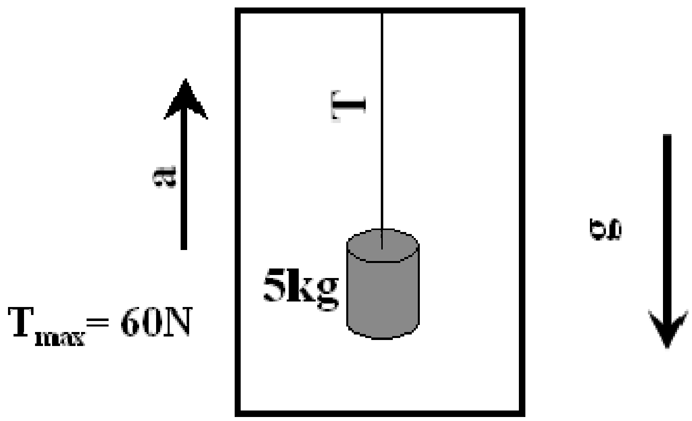
Imagine now that the rope is cut and the block slides a distance D to the bottom of the ramp. Determine the final velocity at the bottom of the ramp.

There are some forces on the block that will cause its acceleration down the ramp. If we could find this acceleration, we could use the velocity-distance kinematic equation (the "timeless" one) to find the velocity at the bottom of the ramp. We can look at the forces on the block and use Newton's Second Law to determine its acceleration.

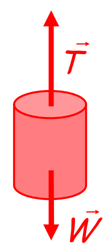
Draw the free body diagram for the block so we can find all the forces on it. There should be two, its weight and a normal force from the ramp. Since we know the acceleration will be along the ramp, it is convenient to choose an axis along that direction. (we chose the x-axis along the ramp in this problem)	
Now we can write Newton's Second Law in each direction. We know the y-acceleration is zero. Since we're after the x-acceleration, let's start with the x-direction.	$\vec{F}_{net} = M\vec{a}$ x-direction: $F_{net,x} = Ma_x$ $N_x + W_x = Ma_x$ $0 + (\cancel{M}g\sin\theta) = \cancel{M}a_x$ $a_x = g\sin\theta$
Now that we know the acceleration, we can use the velocity-distance kinematic equation to find the velocity at the bottom. The block starts from rest at the origin and ends at a displacement of D.	$v^2 = (v_0)^2 + 2a(x - x_0)$ $v^2 = 0 + 2(g\sin\theta)(D - 0)$ $\boxed{v = \sqrt{2gD\sin\theta}}$

DQ6) A mass of 5 kg is suspended from the ceiling of an elevator by an ideal massless rope. The rope will break if the tension T exceeds T_{max} = 60 N.

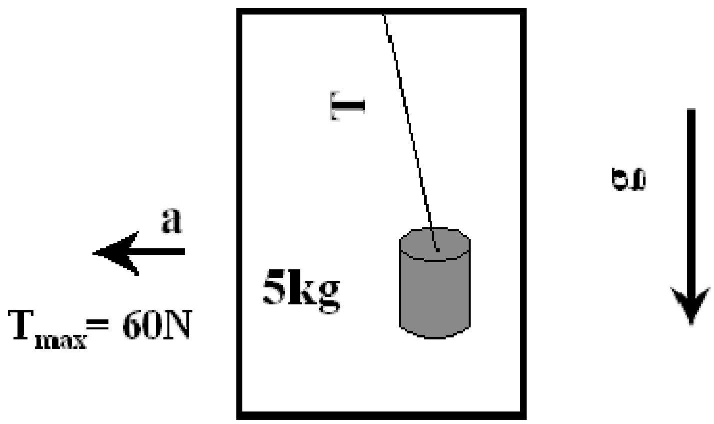
a. The elevator moves upward with the acceleration $\boldsymbol{a}$. What is the maximum vertical acceleration the elevator can have without breaking the rope?



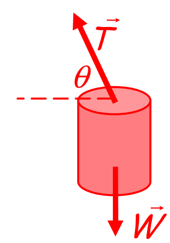
In this problem, the motion is entirely in the y-direction (vertical). We know the tension value of 60 N and are asked for the acceleration. We can use Newton's Second Law for the mass to relate the acceleration to the forces on the mass.

Draw a free body diagram for the mass.	
Use Newton's Second Law in the vertical direction.	$F_{net,y} = Ma_y$ $T_y + W_y = Ma_y$ $T_{max} + (-Mg) = Ma_y$ $a_y = \frac{T_{max}}{M} - g$ $= \frac{60\text{ N}}{5\text{ kg}} - 9.81\frac{\text{m}}{\text{s}^2} = +2.19\frac{\text{m}}{\text{s}^2}$ $= \boxed{2.19\frac{\text{m}}{\text{s}^2}\text{ upward}}$

b. Now the elevator is moving in horizontal direction with the acceleration ***a***. What is the maximum horizontal acceleration the elevator can have without breaking the rope?



In this situation, the acceleration is entirely in the horizontal direction (we'll call it the x-direction). We know the magnitude of the tension but not the direction, so we have two unknowns (the acceleration and the angle of the tension). We'll therefore need to use both the y and x-component equations of Newton's Second Law to find the acceleration. At least we still know that the acceleration in the vertical (y) direction is zero.

Draw a free body diagram for the mass.	
Write the y-component of Newton's Second Law, using the fact that the acceleration in the vertical direction is zero. We can solve this equation for the angle of the string with the horiztonal.	$F_{net,y} = ma_y$ $T_y + W_y = m(0)$ $T_{max}\sin\theta + (-mg) = 0$ $\sin\theta = \frac{mg}{T_{max}}$ $\theta = \sin^{-1}\left(\frac{(5\text{ kg})(9.81\frac{\text{m}}{\text{s}^2})}{60\text{ N}}\right) = 54.8^\circ$
Now we can use the x-component equation of Newton's Second Law with this angle to find the maximum acceleration.	$F_{net,x} = ma_x$ $T_x + W_x = ma_x$ $(T\cos\theta) + 0 = ma_x$ $a_x = \frac{T}{m}\cos\theta$ $= \frac{60\text{ N}}{5\text{ kg}}\cos 54.8^\circ$ $= \boxed{6.92\frac{\text{m}}{\text{s}^2}}$

Formula Sheet

Definitions

Position x

Velocity $v = \frac{dx}{dt}$

Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Constant Acceleration

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Relative Motion

$$\vec{v}_{A,B} = \vec{v}_{A,E} + \vec{v}_{E,B}$$

$$\vec{v}_{E,B} = -\vec{v}_{B,E}$$

Newton's Laws

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

Constants and Conversions

$$g = 9.81\ \tfrac{\text{m}}{\text{s}^2} = 32\ \tfrac{\text{ft}}{\text{s}^2}$$

$$1\ \text{mile} = 1.609\ \text{km}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$