DQ1) Two cannon balls, each with mass 20 kg, are fired from ground level over a flat plateau. Ball A is fired with an initial speed of 250 m/s at an angle of 30 degrees from the horizontal, and Ball B is fired from the same spot with an initial speed of 200 m/s at an angle of 45 degrees from the horizontal.

a) Which ball spends more time in the air? Check with your group members to see if you all agree and how many ways there are to figure out the answer to this question.

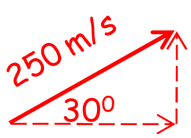
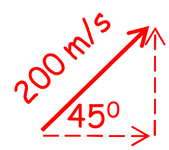
The time in the air is determined by the y component of the motion. Since each ball has the same acceleration (g=9.81 m/s2 downward), the ball with the largest upward velocity will spend the most time in the air. You could also make this conclusion from the velocity-time kinematic equation:

$$v_y = v_{y0} + a_y t$$

$$t_{top} = \frac{v_y - v_{y0}}{a_y} = \frac{(0) - v_{y0}}{(-g)} = \frac{v_{y0}}{g}$$

Since these cannon balls land at the same height they were launched from, the total time for these trajectories is twice the time to the maximum height.

The upshot of all of this is that the cannon ball with the largest upward velocity spends the most time in the air. Consequently, all we have to calculate is the y-component of the launch velocities.

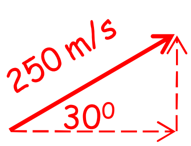
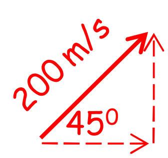
Ball A	Ball B
 $v_{A0,y}$=(250 m/s) sin(30^0)=125 m/s	 $v_{B0,y}$=(200 m/s) sin(45^0)=141 m/s

Since B has the largest upward velocity component, Ball B spends the most time in the air.

Ball B

b) Which ball has the greatest horizontal speed when it lands?

The horizontal speed is unchanged because there is no x-acceleration. Since the final horizontal speed equals the initial, we can just compare the initial x-components of their velocities. The ball with the largest initial x-component of velocity will have the largest final horizontal speed.

Ball A	Ball B
 $v_{A0,x}$=(250 m/s) cos(30^0)=217 m/s	 $v_{B0,x}$=(250 m/s) cos(45^0)=141 m/s

Since Ball A has the largest initial x-component of velocity, Ball A lands with the largest horizontal speed.

Ball A

c) Which ball has the greatest vertical speed when it lands?

From the symmetry of the problem (i.e. landing at the same height they were launched at), the vertical component of the final velocity will be the opposite of the vertical component of the initial velocity. Using the calculations from part a), we see that the two vertical velocities will be

$v_{Af} = -v_{A0} = -125$ m/s

$v_{Bf} = -v_{B0} = -141$ m/s

Ball B therefore has the greatest vertical speed when it lands.

Ball B

d) Which ball travels farther horizontally through the air?

Assuming you didn't calculate the flight time in part a), we need to calculate the flight time for each ball from the vertical motion. Then we can insert that time into the position-time equation for the horizontal motion.

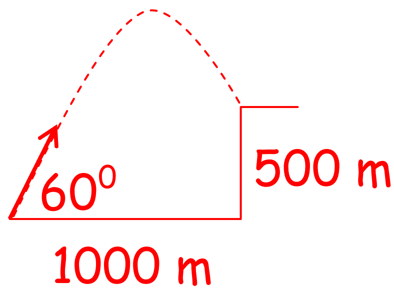
Start by analyzing the motion in the y-dimension to determine the flight time. The ball starts and lands at height zero and has acceleration (-g) (assuming up is positive). The t=0 solution represents the starting situation. We're interested in the later time when the ball hits the ground.	y-dimension $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ $(0) = (0) + (v_0 \sin\theta)t + \frac{1}{2}(-g)t^2$ $0 = t\left(v_0 \sin\theta - \frac{gt}{2}\right)$ $v_0 \sin\theta - \frac{gt}{2} = 0$ $t = \frac{2v_0 \sin\theta}{g}$
We can plug this time into the displacement vs. time for the x-direction to find how far each ball went. The x-component of the acceleration is zero. Here we plug in the flight time found from analyzing the y-direction.	x-dimension $x = x_0 + v_{0x} + \frac{1}{2}a_x t^2$ $x = (0) + (v_0 \cos\theta)t + \frac{1}{2}(0)t^2$ $x = (v_0 \cos\theta)t$ $x = (v_0 \cos\theta)\left(\frac{2v_0 \sin\theta}{g}\right)$ $x = \frac{2(v_0)^2}{g}\sin\theta\cos\theta$
Now we can plug in the numbers for each ball. **Ball A** travels the farthest horizontal distance.	Ball A $x_A = \frac{2(250\,\frac{m}{s})^2}{9.81\,\frac{m}{s^2}}\sin 30^\circ \cos 30^\circ = 5517$ m Ball B $x_B = \frac{2(200\,\frac{m}{s})^2}{9.81\,\frac{m}{s^2}}\sin 45^\circ \cos 45^\circ = 4077$ m

Reflection: Could you have solved part d by only considering the horizontal components of the ball's motion? Talk this over with your group and write down your explanation.

No, the longer air time for ball B is compensated for by a slower x-component of velocity. We don't know ahead of time which factor wins, longer time or larger x-velocity. We actually have to calculate the range.

DQ2) An artillery cannon is aimed at an elevation of 60 degrees above the horizontal.

a) With what speed must it launch a shell to hit a target 1 km away and 500 m above the cannon's altitude?

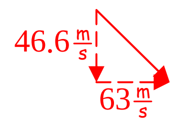
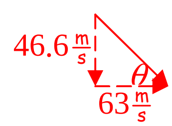


We can find the time to impact (in terms of the launch speed) from the horizontal motion of the shell. We can then substitute this time into the equation describing the y-motion and solve for the launch speed.

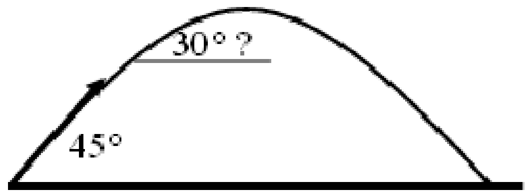
Write the position vs. time for the x-direction and solve for the flight time in terms of the initial velocity.	x direction $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ $x = (0) + (v_0\cos\theta)t + \frac{1}{2}(0)t^2$ $x = (v0\cos\theta)t$ $t = \dfrac{x}{v_0\cos\theta}$
Write the position vs. time for the y-direction and insert the expression for the flight time.	$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ $y = y_0 + (v_0\sin\theta)t + \frac{1}{2}(-g)t^2$ $y = (0) + (v_0\sin\theta)\left(\dfrac{x}{v_0\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{x}{v_0\cos\theta}\right)^2$ $y - x\tan\theta = -\dfrac{gx^2}{2(v_0)^2\cos^2\theta}$ $v_0 = \sqrt{\dfrac{gx^2}{2(x\tan\theta - y)\cos^2\theta}}$
Insert the numerical parameters from the problem statement.	$v_0 = \sqrt{\dfrac{(9.81\,\frac{\text{m}}{\text{s}^2})(1000\text{ m})^2}{2((1000\text{ m})\tan 60^\circ - (500\text{ m}))(\cos 60^\circ)^2}}$ $= \sqrt{\dfrac{9.81\times 10^6\,\frac{\text{m}^2}{\text{s}^2}}{2((1000\text{ m})(1.732) - (500\text{ m}))(0.5)^2}}$ $= \boxed{126\,\frac{\text{m}}{\text{s}}}$

b) At this launch speed (also known as muzzle speed), with what velocity (speed and direction) will the shell strike the target?

Now that we know the initial launch velocity of the cannonball, we can find the time of flight from the motion in the x-direction. We know the x-velocity remains constant. We can find the y-velocity at impact from the velocity-time equation in the y-direction.

We know the initial velocity, so we can find the time of flight from the motion in the x-direction (see the first box of part a)	$t = \frac{x}{v_0 \cos\theta}$ $= \frac{1000\ \text{m}}{(126\,\frac{\text{m}}{\text{s}})\cos 60^\circ} = 15.87\ \text{s}$
The x-velocity remains constant, but the y-velocity changes due to the acceleration due to gravity.	$v_x = v_{0x} + a_x t$ $= v_0 \cos\theta + (0)t$ $= (126\,\frac{\text{m}}{\text{s}})\cos 60^\circ = 63\,\frac{\text{m}}{\text{s}}$ $v_y = v_{0y} + a_y t$ $= v_0 \sin\theta + (-g)t$ $= (126\,\frac{\text{m}}{\text{s}})\sin 60^\circ - (9.81\,\frac{\text{m}}{\text{s}^2})(15.87\ \text{s})$ $= -46.6\,\frac{\text{m}}{\text{s}}$
The final velocity magnitude is the vector sum of the x- and y-component of the final velocity. We can use the Pythagorean Theorem to find the final speed from the two components.	 $v = \sqrt{v_x^2 + v_y^2}$ $= \sqrt{(+63\,\frac{\text{m}}{\text{s}})^2 + (-46.6\,\frac{\text{m}}{\text{s}})^2} = \boxed{78.4\,\frac{\text{m}}{\text{s}}}$
We can use trigonometry to find the direction of the final velocity	 $\tan\theta = \frac{46.6\,\frac{\text{m}}{\text{s}}}{63\,\frac{\text{m}}{\text{s}}} = .740$ $\theta = \tan^{-1}(0.740)$ $= \boxed{36.5^\circ \text{ below the horizontal}}$

DQ3) A projectile is shot out at 45° with respect to the horizon with an initial velocity of $V_x = V_y = V_{comp}$. When is the earliest time, t, that the velocity vector makes an angle of 30° with respect to the horizontal? Try this problem on your own first, then ask your group about it and check your answers.



a) $t = V_{comp} / (g \sin 30°)$
b) $t = V_{comp} (1 - \sin 30°) / g$
c) $t = V_{comp} \tan 30° / g$
d) $t = V_{comp} (1 - \tan 30°) / g$
e) $t = V_{comp} / (g \tan 30°)$

We know that the projectile will undergo an acceleration due to gravity of g downward. This will change the y-component of the projectile's velocity but will not affect its x-component. This means the velocity angle with the horizontal will change. We have three relationships we can use:

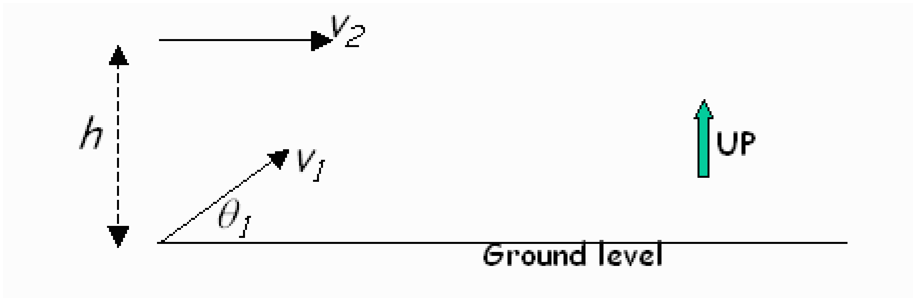
1) the final velocity makes an angle of 30° with the horizontal

2) the x-component of the final velocity is unchanged

3) the y-component of the final velocity can be found from the velocity-time kinematic equation

The projectile velocity changes according to the constant acceleration kinematic equations	$v_y = v_{0y} + a_y t$ $v_y = V_{comp} + (-g)t$ $v_x = v_{0x} + a_x t$ $= V_{comp} + (0)t$ $v_x = V_{comp}$
The final velocity makes an angle of 30° with the horizontal, giving us a relationship between the x- and y-components of the final velocity.	$v_y = v_x \tan 30^\circ$ $v_y = (V_{comp}) \tan 30^\circ$
We can insert this relationship for v_y into the kinematic equation involving the time.	$v_y = V_{comp} - gt$ $(V_{comp} \tan 30^\circ) = V_{comp} - gt$ $t = \dfrac{V_{comp} - (V_{comp} \tan 30^\circ)}{g} = V_{comp} \dfrac{1 - \tan 30^\circ}{g}$

DQ4) Consider two projectiles that are fired at the same time. Projectile 1 is fired from ground level with velocity v_1 at an angle θ_1, while projectile 2 is fired at velocity v_2 from a height h above the ground, as shown in the diagram below.



a. Which parameters (v_1, θ_1, v_2, h) do you need to know to be able to determine which projectile lands first?

The time in the air is determined entirely by the vertical motion, namely the initial height and the initial y-velocity. We'll therefore need the height h of the second projectile. We already know its vertical velocity component (zero) because it's fired horizontally. We know the initial height of the first projectile (zero). To find its vertical velocity component, we'll need the magnitude of its initial velocity v_1 and the firing angle θ_1. We therefore need to know:

H, v_1, and θ_1

b. Find the equation for v_1 in terms of the variables above and other constants (g, ...) in the situation where the two balls spend the same amount of time in the air.

We know that the two projectiles spend the same amount of time in the air. We therefore need to express this time in terms of the parameters stated in part a. We can use the kinematic equations with the known gravitational acceleration (g downward) to do this. We know the initial and final positions of each of the objects, their initial velocities, and their accelerations. Since we're looking for the time from distance information, the distance-time kinematic equation is a good place to start.

The first projectile is launched from zero height with speed v_1 at an angle of θ_1. We of course get two solutions for the flight time. The t=0 solutions corresponds to our launch time, when the first projectile also has a height of zero.	$y_1 = y_{10} + v_{1y}t + \frac{1}{2}a_y t^2$ $(0) = (0) + (v_1 \sin\theta)t_1 + \frac{1}{2}(-g)t_1^2$ $0 = t_1\left(v_1 \sin\theta - \frac{gt_1}{2}\right)$ $t_1 = 0$ OR $\left(v_1 \sin\theta - \frac{gt_1}{2}\right) = 0$ $t_1 = \frac{2v_1 \sin\theta}{g}$
The second projectile is launched from height h with speed v_2 horizontally (i.e. with $v_{0y} = 0$)	$y_2 = y_{20} + v_{2y}t_2 + \frac{1}{2}a_y t_2^2$ $(0) = (h) + (0)t_2 + \frac{1}{2}(-g)t_2^2$ $t_2 = \sqrt{\frac{2h}{g}}$

The two balls spend the same amount of time in the air. We can insert the expressions for the flight times found above. We can simplify the answer by bringing the factors of 2 and g under the radical sign.	$t_1 = t_2$ $\left(\frac{2v_1 \sin\theta}{g}\right) = \left(\sqrt{\frac{2h}{g}}\right)$ $v_1 = \frac{g}{2\sin\theta}\sqrt{\frac{2h}{g}}$ $= \frac{1}{\sin\theta}\sqrt{\frac{g^{2}}{2^{2}}\frac{2h}{g}}$ $\boxed{v_1 = \frac{1}{\sin\theta}\sqrt{\frac{gh}{2}}}$

DQ5) (challenge problem) An elevator accelerates upward from rest, accelerating steadily from rest to 6 m/s in 6 seconds. Three seconds into the acceleration, a bolt falls off the bottom of the elevator.

a) What is the distance between the bolt and the bottom of the elevator after the 6 seconds are up?

We know the acceleration of the bolt (gravitational acceleration g downward). If we can find the acceleration of the elevator, we can use the distance-time constant acceleration kinematic equation for the elevator and the bolt separately. We can then take the difference between them to find the separation as a function of time.

First we must find the acceleration of the elevator (subscript E). We know its initial and final velocities, as well as the time the motion takes. The velocity-time kinematic equation is therefore probably most useful.	$v_E = v_{E0} + a_E t$ $a_E = \frac{v_E - v_{E0}}{t} = \frac{(6\,\frac{m}{s}) - (0)}{6\ s} = 1\,\frac{m}{s^2}$
We can write the elevator's vertical position as a function of time using the distance-time kinematic equation.	$y_E(t) = y_{E0} + v_{E0y}t + \frac{1}{2}a_{Ey}t^2$ $= (0) + (0)t + \frac{1}{2}a_E t^2$ $y_E(t) = \frac{1}{2}a_E t^2$
To analyze the bolt's motion, it's easiest to reset the clock to zero (new time t') when the bolt falls off. Both the elevator and the bolt will have the same initial position (y_0) and initial velocity (v_{y0}). Only their accelerations will differ (+1m/s^2 for the elevator, -9.81 m/s^2 for the bolt)	$y_E(t') = y_0 + v_{0y}t' + \frac{1}{2}a_{Ey}(t')^2$ $y_B(t') = y_0 + v_{0y}t' + \frac{1}{2}a_{By}(t')^2 = y_0 + v_{0y}t' + \frac{1}{2}(-g)(t')^2$
Take the difference in positions. The only terms that survive come from their different accelerations.	$y_E(t') - y_B(t') = \left(\cancel{y_0} + \cancel{v_{0y}t'} + \frac{1}{2}a_{Ey}(t')^2\right) - \left(\cancel{y_0} + \cancel{v_{0y}t'} - \frac{1}{2}g(t')^2\right)$ $= \frac{1}{2}(a_{Ey} + g)(t')^2$
The bolt falls for 3 seconds after being released, so t'=3 seconds.	$t' = 3\ s$ $y_E(t') - y_B(t') = \frac{1}{2}\left((+1\,\frac{m}{s^2}) + (9.81\,\frac{m}{s^2})\right)(3\ s)^2$ $= \boxed{48.6\ m}$

b) What would your answer be if, 3 seconds into the motion, someone on the elevator instead threw the bolt horizontally at a speed of 3 m/s?

Giving the bolt an x-velocity as well does not affect the vertical motion. The final y-position is therefore unchanged. The horizontal distance traveled by the bolt would therefore be (3 m/s)(3 s) = 9 m. The total distance between the bolt and elevator at the end of the total 6 seconds is therefore $\sqrt{(48.6\text{ m})^2 + (9\text{ m})^2} = 49.4\text{ m}$.