Discussion Session 2

PHYS 100

Constant Acceleration/Relative Motion

Week 03

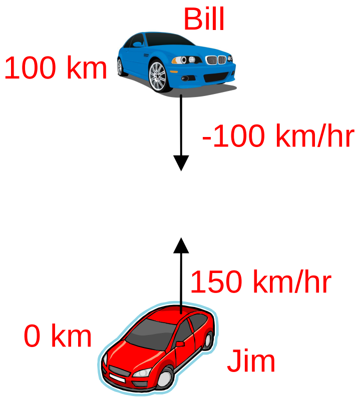
The Plan

Today you will work with your group explore the ideas of reference frames (i.e. relative motion) and motion with constant acceleration. You'll begin by visualizing some situations using diagrams or graphs to get a feel for the problem. Then you'll solve these problems using the basic kinematic ideas you've learned in PreLecture and practiced on your homework.

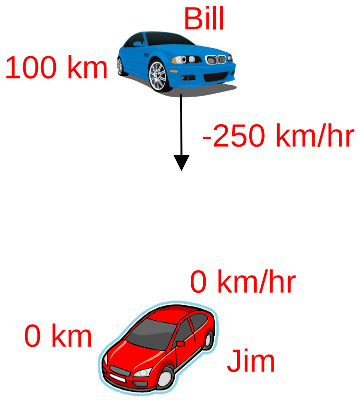
After this initial practice, your group will solve some numerical and some symbolic problems to practice representing physical relationships using the constant acceleration kinematic ideas. At the end, there's a chance for your group to test your understanding by finding the mistake(s) in an already worked our solution.

DQ1) Jim drives his Viper 150 km/hr northward. Bill, who is 100 km north of Jim, drives his Corvette southward at 100 km/hr. When will they meet?

a) Talk with your group to generate a sketch of this situation from the perspective of someone standing by the roadside (i.e. in the “Earth’s reference frame”). Make sure you label all displacements and velocities.



b) Now talk together to generate a sketch of this situation from Jim’s perspective (i.e. in “Jim’s reference frame”). Again, make sure you label all displacements and velocities.



To determine the velocities in Jim’s frame, remember that in Jim’s frame, Jim appears at rest. To go from 150 km/hr in the earth’s frame to rest implies we subtract Jim’s velocity from any velocity in the Earth frame:

$$\vec{v}_{\text{Jim,Jim}} = \vec{v}_{\text{Jim,Earth}} - \vec{v}_{\text{Jim,Earth}} = \left(150\,\tfrac{\text{km}}{\text{hr}}\right) - \left(150\,\tfrac{\text{km}}{\text{hr}}\right) = 0\,\tfrac{\text{km}}{\text{hr}}$$

$$\vec{v}_{\text{Bill,Jim}} = \vec{v}_{\text{Bill,Earth}} - \vec{v}_{\text{Jim,Earth}} = \left(-100\,\tfrac{\text{km}}{\text{hr}}\right) - \left(150\,\tfrac{\text{km}}{\text{hr}}\right) = -250\,\tfrac{\text{km}}{\text{hr}}$$

c) In which frame does the problem look simpler? Discuss this question with your group, writing down some of the factors that go into deciding.
In Jim’s frame, only one object is moving, while in the earth’s frame both objects are moving. Jim’s frame make the motion appear simpler.

d) Working as a group, solve the problem in your chosen simplest frame of reference. How long does it take for the two cars to meet?

In Jim’s frame, the Bill travels 100 km at 250 km/hr, so they should meet in $\frac{100\text{ km}}{250\,\frac{\text{km}}{\text{hr}}} = 0.4\text{ hr}$.

To get this systematically from the kinematic equations, we would use the known displacement, velocity, and acceleration of Bill and solve for the time:

$$x_{\text{Bill}} = x_{0,\text{Bill}} + v_{0,\text{Bill}}t + \tfrac{1}{2}a_{\text{Bill}}t^2$$

$$(0\text{ km}) = (100\text{ km}) + \left(-250\,\tfrac{\text{km}}{\text{hr}}\right)t + \tfrac{1}{2}\left(0\,\tfrac{\text{km}}{\text{hr}^2}\right)t^2$$

$$-100\text{ km} = \left(-250\,\tfrac{\text{km}}{\text{hr}}\right)t$$

$$t = \frac{-100\text{ km}}{-250\,\frac{\text{km}}{\text{hr}}} = 0.4\text{ hr}$$

If you were to solve this problem in the Earth's frame, you would need to write the position as a function of time for each car, then set their positions equal to find the time that they meet:

$$x_{\text{Bill}}(t) = x_{0,\text{Bill}} + v_{0,\text{Bill}} + \tfrac{1}{2}a_{\text{Bill}}t^2 \qquad x_{\text{Jim}}(t) = x_{0,\text{Jim}} + v_{0,\text{Jim}} + \tfrac{1}{2}a_{\text{Jim}}t^2$$

$$= (100\ \text{km}) + \left(-100\,\tfrac{\text{km}}{\text{hr}}\right)t + \tfrac{1}{2}\left(0\,\tfrac{\text{km}}{\text{hr}^2}\right)t^2 \qquad = (0\ \text{km}) + \left(150\,\tfrac{\text{km}}{\text{hr}}\right)t + \tfrac{1}{2}\left(0\,\tfrac{\text{km}}{\text{hr}^2}\right)t^2$$

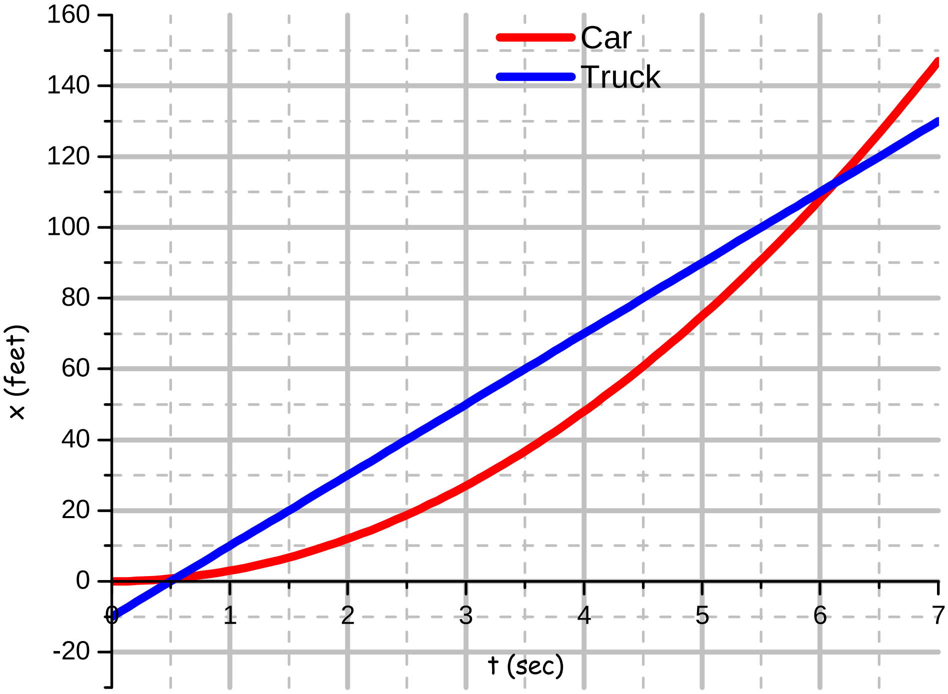
$$x_{\text{Bill}}(t) = (100\ \text{km}) - \left(100\,\tfrac{\text{km}}{\text{hr}}\right)t \qquad x_{\text{Jim}}(t) = \left(150\,\tfrac{\text{km}}{\text{hr}}\right)t$$

Setting the two positions equal allows us to find the meeting time:

$$\begin{aligned} x_{\text{Jim}}(t_{\text{meet}}) &= x_{\text{Bill}}(t_{\text{meet}}) \\ v_{\text{Jim}}t_{\text{meet}} &= d - v_{\text{Bill}}t_{\text{meet}} \\ (v_{\text{Jim}} + v_{\text{Bill}})t_{\text{meet}} &= d \\ t_{\text{meet}} &= \frac{d}{v_{\text{Jim}} + v_{\text{Bill}}} = \frac{100\ \text{km}}{150\,\frac{\text{km}}{\text{hr}} + 100\,\frac{\text{km}}{\text{hr}}} = \frac{100\ \text{km}}{250\,\frac{\text{km}}{\text{hr}}} = 0.4\ \text{hr} \end{aligned}$$

DQ2) At the instant a traffic light turns green, a car that has been waiting at the intersection starts moving at a constant acceleration of 6 ft/s^2. At the same instant a truck is 10 feet behind the car, traveling with a constant velocity of 20 ft/s.

a) Working with your group, plot the motion of the car and the truck on the axes below. Assume x=0 is the initial position of the car. Also be sure to plot the motion over the full seven seconds.



These plots came from using the general displacement vs. time expression for constant acceleration kinematics:

Car

$$x_c(t) = x_{c0} + v_{c0}t + \tfrac{1}{2}a_c t^2$$
$$= (0) + (0)t + \tfrac{1}{2}\left(6\tfrac{\text{ft}}{\text{s}^2}\right)t^2$$
$$= \left(3\tfrac{\text{ft}}{\text{s}^2}\right)t^2$$

Truck

$$x_T(t) = x_{T0} + v_{T0}t + \tfrac{1}{2}a_T t^2$$
$$= (-10\text{ m}) + \left(20\tfrac{\text{ft}}{\text{s}}\right)t + \tfrac{1}{2}(0)t^2$$
$$= (-10\text{ m}) + \left(20\tfrac{\text{ft}}{\text{s}}\right)t$$

The car's graph should therefore be a parabola through the origin, while the truck's should be a straight line of slope 20 ft/s going through the point (0 seconds, -10 meters). You can then plot the graphs with a few representative points

b) How many times do the two automobiles pass one another? At what time(s) does this occur? (Use equations to find the values then check them with the graphs)

From the graph, it appears the two automobiles pass each other twice. We can get these times from setting their positions equal and solving for the meeting times:

$$x_c(t_{meet}) = x_{\mathrm{T}}(t_{meet})$$

$$\left(3\,\tfrac{\text{ft}}{\text{s}^2}\right)t_{meet}{}^2 = \left(-10\ \text{ft}\right) + \left(20\,\tfrac{\text{ft}}{\text{s}}\right)t_{meet}$$

$$\left(3\,\tfrac{\text{ft}}{\text{s}^2}\right)t_{meet}{}^2 + \left(-20\,\tfrac{\text{ft}}{\text{s}}\right)t_{meet} + \left(10\ \text{ft}\right) = 0$$

$$\begin{aligned} t_{meet} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-\left(-20\,\tfrac{\text{ft}}{\text{s}}\right) \pm \sqrt{\left(-20\,\tfrac{\text{ft}}{\text{s}}\right)^2 - 4\left(3\,\tfrac{\text{ft}}{\text{s}^2}\right)\left(10\ \text{ft}\right)}}{2\left(3\,\tfrac{\text{ft}}{\text{s}^2}\right)} \\ &= \frac{\left(20\,\tfrac{\text{ft}}{\text{s}}\right) \pm \left(16.733\,\tfrac{\text{ft}}{\text{s}}\right)}{6\,\tfrac{\text{ft}}{\text{s}^2}} \\ &= \boxed{0.54\ \text{s}}\ \text{and}\ \boxed{6.12\ \text{s}} \end{aligned}$$

Note that we had to use the quadratic formula to solve the quadratic equation resulting from setting the car and truck positions equal to one another. These two times are just after 0.5 seconds and just after 6 seconds. They agree nicely with the intersections in the plot above.

c) Compare the velocities of the two automobiles when they pass. Check with your group to see if you all agree and if your answers make sense in this physical situation.

Physically, we'd expect the initially faster-moving truck to pass the car. Later, the now faster-moving car should catch the truck. We'd therefore expect the truck to have a faster speed at their first meeting, then the car have a faster speed at their second meeting. The graphs agree with this idea; the truck's slope (velocity) is steeper at their first meeting, but the car's slop is steeper at their second meeting. Numerically, the truck's velocity never changes (always 20 ft/s). The car's velocity is given by $v_c = v_{c0} + a_c t = (0) + \left(6\,\tfrac{\text{ft}}{\text{s}^2}\right)t$. The car's speeds at their meeting times are therefore 3.8 ft/s and 37 ft/s. As expected, the car is going slower at their first meeting and faster at their second.

DQ3) You are designing an apparatus to test the acceleration tolerances of products. Your responsibility is a fastener that connects a launch sled to some guide wires. To reduce weight, you have been forced to use an adhesive that can withstand a maximum acceleration of 10,000 m/s² in this application (already accounting for safe design tolerances).

a) What is the maximum velocity of the launch sled if the acceleration path is 0.5 m long and the sled starts from rest?

We are neither given nor asked for any time, so let's use the "timeless" equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$= (0)^2 + 2\left(10{,}000\,\tfrac{\text{m}}{\text{s}^2}\right)(0.5\text{ m} - 0\text{ m})$$

$$v = \sqrt{10{,}000\,\tfrac{\text{m}^2}{\text{s}^2}} = \boxed{100\,\tfrac{\text{m}}{\text{s}}}$$

b) How long would the launch take?

We know the initial velocity (zero), the final velocity (100 m/s), and the acceleration (10^4 m/s^2):

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{\left(100\,\tfrac{\text{m}}{\text{s}}\right) - \left(0\,\tfrac{\text{m}}{\text{s}}\right)}{10{,}000\,\tfrac{\text{m}}{\text{s}^2}} = 0.01\text{ s}$$

DQ4) A boat is travelling at maximum speed through still water straight toward a dock. Using full reverse, the daredevil boater knows how long it takes to stop his boat at this speed. How close can he get to the dock before throwing the engines in reverse (and still avoid hitting the dock)? Talk with your group to define some symbols and obtain an equation relating the shortest braking distance to the other known quantities in the problem.

Let's define symbols for the quantities that the boater knows/wants to know:

v_{max} = maximum boat speed
v = final boat speed $= 0$
t_{stop} = time it takes to stop the boat
d_{min} = minimum stopping distance needed

We could use either of the equations $x = x_0 + v_0 t + \tfrac{1}{2}at^2$, $v^2 = v_0^2 + 2a(x - x_0)$ if we know the acceleration. We'll use "velocity-time" equation to figure out the acceleration from the stopping time. Then we can use one of the other two to get the distance over which the stopping happens.

Using the "velocity-time" equation:

$$v = v_0 + at$$

$$a = \frac{v - v_0}{t} = \frac{0 - v_{\text{max}}}{t_{\text{stop}}} = -\frac{v_{\text{max}}}{t_{\text{stop}}}$$

Now that we know the acceleration, let's find the stopping distance, assuming the boat starts at zero position:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$d_{\text{min}} = (0) + v_{\text{max}} t_{\text{stop}} + \tfrac{1}{2}\left(-\frac{v_{\text{max}}}{t_{\text{stop}}}\right) t_{\text{stop}}^2$$

$$d_{\text{min}} = v_{\text{max}} t_{\text{stop}} - \frac{v_{\text{max}} t_{\text{stop}}}{2} = \boxed{\frac{v_{\text{max}} t_{\text{stop}}}{2}}$$

Tatiana walks steadily all the way up a stalled escalator at a constant speed taking 90 seconds to do so. After the escalator is fixed it ascends at a constant speed, and she rides up the same distance without walking in 60 seconds. How much time would it take her to walk up the moving escalator if she walks at the same rate up the moving escalator as she did when the escalator was broken?

For this problem we know the time it takes to go the escalator distance when walking or riding are asked to find the time for walking while the escalator is moving. We don't know the length of the escalator or the walking speeds. Let's start by defining some symbols:

$t_{\text{walk}} = 90\ s =$ time to walk up the stalled escalator

$t_{\text{ride}} = 60\ s =$ time to ride up the moving escalator without walking

$L =$ the (unknown) length of the escalator

$v_{\text{walk}} =$ (unknown) Tatiana's walking speed

$v_{\text{ride}} =$ (unknown) Tatiana's speed while riding without walking

We know some physical relationships among these quantities:

(1) velocity is distance over time
(2) the walk+ride speed will be the sum of the walk and ride speeds

Starting with the second relationship, $v_{\text{walk+ride}} = v_{\text{walk}} + v_{\text{ride}}$, we can use the first one to get an expression for the total time for walking and riding together: $t_{\text{tot}} = \frac{L}{v_{\text{walk+ride}}} = \frac{L}{v_{\text{walk}} + v_{\text{ride}}}$.

The walking and riding speeds can also be written in terms of L and known times: $v_{\text{walk}} = \frac{L}{t_{\text{walk}}}$, $v_{\text{ride}} = \frac{L}{t_{\text{ride}}}$

Plugging these into the total time expression gives us the total time to walk and ride together.

$$
\begin{aligned}
t_{\text{tot}} &= \frac{L}{v_{\text{walk}} + v_{\text{ride}}} \\
&= \frac{L}{\frac{L}{t_{\text{walk}}} + \frac{L}{t_{\text{ride}}}} = \frac{\cancel{L}}{\cancel{L}\left(\frac{1}{t_{\text{walk}}} + \frac{1}{t_{\text{ride}}}\right)} \\
&= \frac{1}{\frac{1}{t_{\text{walk}}} + \frac{1}{t_{\text{ride}}}} = \frac{1}{\frac{1}{90\ s} + \frac{1}{60\ s}} = \boxed{36\ s}
\end{aligned}
$$

A worked-out solution to a problem is presented below but contains at least one error. Read over the solution and discuss with your group what the possible errors are.

Situtation: The local Coast Guard unit is interested in purchasing a new boat. They want to be able to catch drug-runners rounding a point 2 nautical miles away from their dock. The drug boats can travel at 20 nautical miles/hour (knots). The Coast Guard boat under consideration can travel at 45 knots but takes 30 seconds to get up to full speed. Could this boat catch the drug-runners before they reach international waters 12 nautical miles away from the dock?

Strategy: Write functions for the position of each of the boats as a function of time. Find the time at which the Coast Guard (CG) boat catches the drug boat (DB), then find the location where it happened. If the location is less than 12 nautical miles, the Coast Guard boat can catch the drug boat.

Write the drug boat's position as a function of time. It has no acceleration and starts 2 nautical miles (d) from the dock	$x_{DB}(t) = d + v_{DB}t$
We don't know the Coast Guard boat's acceleration (a_{CG}). We can find it from its maximum velocity ($v_{CG,\max}$).	$v_{CG,\max} = v_{CG,0} + a_{CG}t$ $= 0 + a_{CG}t$ $a_{CG} = \frac{v_{CG,\max}}{t}$ This should be the 30s acceleration time (t_{accel})
Write the Coast Guard boat's equation as a function of time. It has the acceleration above and starts from rest.	$x_{CG}(t) = \frac{1}{2}\left(\frac{v_{CG,\max}}{t}\right)t^2$ $= \frac{v_{CG,\max}t}{2}$ This equation assumes the CG boat accelerates the whole time (rather than just 30 s)
Set their positions equal to find the time the boats meet.	$x_{CG}(t_{meet}) = x_{DB}(t_{meet})$ $\frac{v_{CG,\max}t_{meet}}{2} = d + v_{DB}t_{meet}$ $v_{CG,\max}t_{meet} = 2d + 2v_{DB}t_{meet}$ $(v_{CG,\max} - 2v_{DB})t_{meet} = 2d$ $t_{meet} = \frac{2d}{v_{CG,\max} - 2v_{DB}} = \frac{2(2\text{ nmiles})}{\left(45\,\frac{\text{nmiles}}{\text{hr}}\right) - 2\left(20\,\frac{\text{nmiles}}{\text{hr}}\right)}$ $= 0.8\text{ hours}$
Use this time to find out where the boats meet.	$x_{DB}(t_{meet}) = d + v_{DB}t_{meet}$ $= (2\text{ nmiles}) + \left(20\,\frac{\text{nmiles}}{\text{hr}}\right)(0.8\text{ hr})$ $= 18\text{ nmiles} > 12\text{ nmiles}$ $\Rightarrow$ cannot catch drug boat!

In case you're curious, here's the correct solution:

Write the drug boat's position as a function of time. It has no acceleration and starts 2 nautical miles (d) from the dock	$x_{DB}(t) = d + v_{DB}t$
We don't know the Coast Guard boat's acceleration (a_{CG}). We can find it from its maximum velocity ($v_{CG,\max}$) and its acceleration time (t_{accel}).	$v_{CG,\max} = v_{CG,0} + a_{CG}t_{\text{accel}}$ $= 0 + a_{CG}t_{\text{accel}}$ $a_{CG} = \frac{v_{CG,\max}}{t_{\text{accel}}}$
Find where the CG boat is at the end of its acceleration. Write the Coast Guard boat's equation as a function of time after the acceleration is finished.	$x_{CG,\text{at speed}} = \frac{1}{2}\left(\frac{v_{CG,\max}}{t_{\text{accel}}}\right)t_{\text{accel}}^{2}$ $= \frac{v_{CG,\max}t_{\text{accel}}}{2}$ $x_{CG}(t) = \overbrace{\frac{v_{CG,\max}t_{\text{accel}}}{2}}^{\text{Position at end of acceleration}} + v_{CG,\max}\overbrace{(t - t_{\text{accel}})}^{\text{Time since acceleration finished}}$ $= v_{CG,\max}t - \frac{v_{CG,\max}t_{\text{accel}}}{2}$
Set their positions equal to find the time the boats meet.	$x_{CG}(t_{\text{meet}}) = x_{\text{DB}}(t_{\text{meet}})$ $v_{CG,\max}t - \frac{v_{CG,\max}t_{\text{accel}}}{2} = d + v_{\text{DB}}t_{\text{meet}}$ $v_{CG,\max}t_{meet} = d + v_{DB}t_{meet} + \frac{v_{CG,\max}t_{\text{accel}}}{2}$ $(v_{CG,\max} - v_{DB})t_{meet} = d + \frac{v_{CG,\max}t_{\text{accel}}}{2}$ $t_{meet} = \frac{d + \frac{v_{CG,\max}t_{\text{accel}}}{2}}{(v_{CG,\max} - v_{DB})}$ $= \frac{(2\text{ nmiles}) + \frac{\left(45\,\frac{\text{nmiles}}{\text{hr}}\right)\left(30\text{s} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{1\text{ hr}}{60\text{ min}}\right)}{2}}{\left(45\,\frac{\text{nmiles}}{\text{hr}}\right) - \left(20\,\frac{\text{nmiles}}{\text{hr}}\right)}$ $= 0.0875$ hours
Use this time to find out where the boats meet.	$x_{DB}(t_{meet}) = d + v_{DB}t_{meet}$ $= (2\text{ nmiles}) + \left(20\,\frac{\text{nmiles}}{\text{hr}}\right)(0.0875\text{ hr})$ $= 3.75\text{ nmiles} < 12\text{ nmiles}$ $\Rightarrow$ EASILY CATCHES drug boat!

Formula Sheet

Definitions

Position x

Velocity $v = \frac{dx}{dt}$

Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Constant Acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0{}^2 + 2a\left(x - x_0\right)$$

Constants and Conversions

$$g = 9.81\ \tfrac{\text{m}}{\text{s}^2} = 32\ \tfrac{\text{ft}}{\text{s}^2}$$

$$1\ \text{mile} = 1.609\ \text{km}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$