Discussion Session 1

PHYS 100

Kinematic Definitions

Week 02

The Plan

This week your group will explore two ways to represent motion. One way is to talk about the position, velocity, and acceleration of an object in words, numbers, or symbols. Another is to represent motion using graphs of position, velocity, or acceleration as functions of time.

We'll begin with some descriptions of motion in words, from which your group will generate a graph of the motion. After you come to a consensus, you'll use a computer simulation called *The Moving Man* to explore how the motion and the graphs are related. Play with the simulator to see what parts of the graph correspond to how the motion looks.

Finally, you'll use the ideas you've played with on the simulator to interpret some graphs I give you. Your group will translate these graphs into a description of the motion in prose.

Below is a description of some motion in words. Talk with your group about what the motion would look like if you were watching it happen.

Motion 1) I walk 2.5 m/s eastward for 2 seconds and then 2.5 m/s westward for 2 seconds. All displacements are measured from my starting position.

a. What can you say about the direction of my final displacement vector (circle one)?

points eastward points westward [✓ is zero]

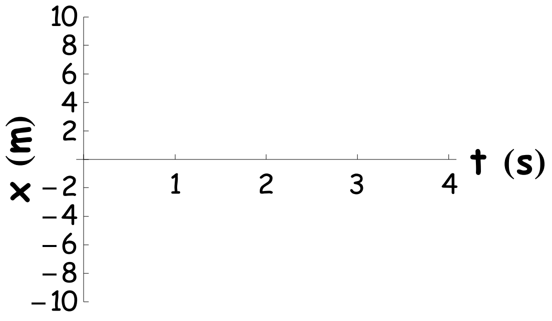
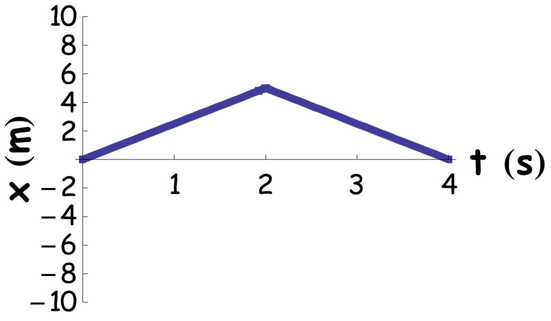
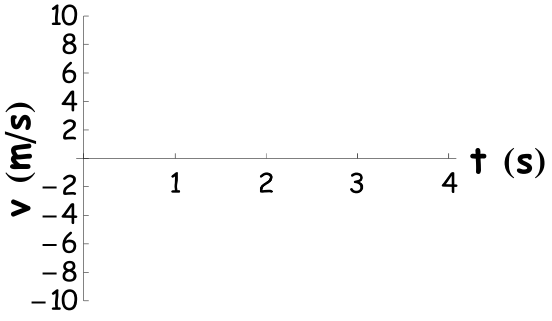
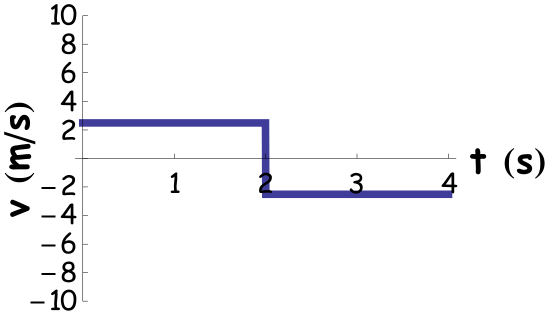
I spend equal time traveling east at constant speed and west at constant speed. The second displacement is therefore equal to the opposite of the first, and I end up where I started.

b. What is the average value of my speed?

The speed is the magnitude of velocity. In this case it is always 2.5 m/s, so the average value is just 2.5 m/s. Mathematically:

$$\text{average } |\vec{v}| = \frac{\left|+2.5\frac{m}{s}\right|(2s)+\left|-2.5\frac{m}{s}\right|(2s)}{(2s)+(2s)} = \left(\frac{2.5\times2+2.5\times2}{4}\right)\frac{m}{s} = \frac{10m}{4s} = 2.5\frac{m}{s}$$

c. Talk with your group about what a position vs. time plot would look like for this motion. Sketch a "prediction" for what this graph should look like below. (don't worry about the "simulation" column for now)

Group Prediction	Simulation Result
	
	

Motion 2) I accelerate eastward from rest at a constant rate of 2.5 m/s^2 for 2 seconds, and then I accelerate westward at a constant rate of 2.5 m/s^2 for 2 more seconds.

a. What is the direction of my final displacement vector (circle one)?

[✓ points eastward] points westward is zero

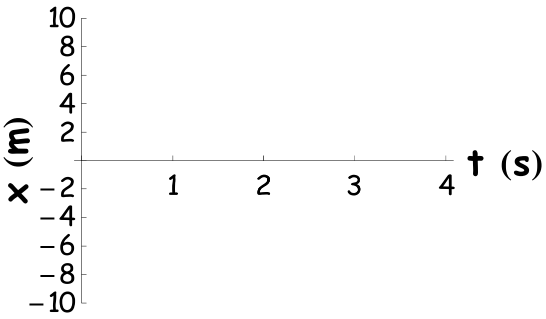
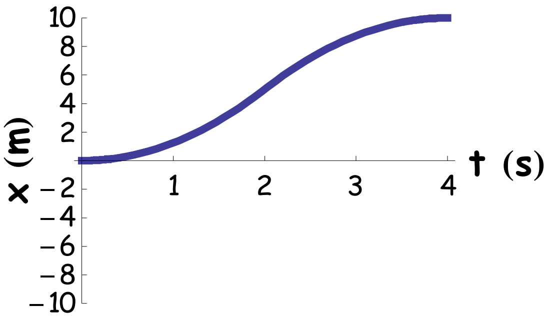
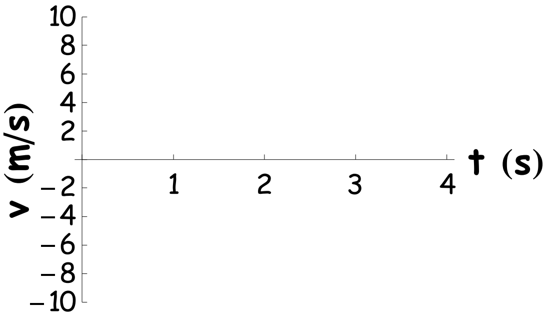
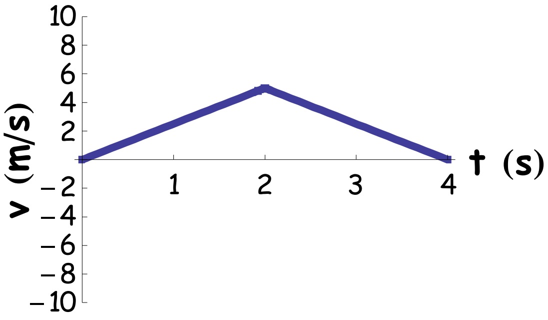
My velocity grows faster eastward (acceleration along velocity) for 2 seconds. It then continues eastward but slows down (acceleration against velocity) for 2 more seconds. Since my velocity is always eastward, my displacement will end up being eastward.

b. What is the value of my instantaneous velocity after 2 seconds? After 4 seconds?

I start from rest and gain 2.5 m/s every second, so after 2 seconds I should be going $\left(0\frac{m}{s}\right)+\left(2.5\frac{m}{s^2}\right)(2s)=\boxed{5\frac{m}{s}}$.

In the next two seconds, I start with 5 m/s eastward and the acceleration points against my velocity. I therefore lose 2.5 m/s every second, giving a final velocity of $\left(5\frac{m}{s}\right)+\left(-2.5\frac{m}{s^2}\right)(2s)=\left(5\frac{m}{s}\right)+\left(-5\frac{m}{s}\right)=\boxed{0\frac{m}{s}}$

c. Talk with your group about what a position vs. time plot would look like for this motion. Sketch a "prediction" for what this graph should look like below. (don't worry about the "simulation" column for now)

Group Prediction	Simulation Result
	
	

Simulation Exploration

Now your group gets a chance to explore these two motions using a simulation from the University of Colorado called the "Moving Man". Your entire group should go to one of the computers in the back of the room and log in using the following ID:

username = Phys100student
password = Phys100student

On the desktop, you'll find an icon with the name "Moving Man". Double click this icon to start the simulation.
(NOTE: you can access this simulator online by pointing your browser to
http://phet.colorado.edu and searching for "moving man")

The simulation will load after a few moments. As you work through the following activities ***take turns at the controls*** so everyone gets a chance to play with the simulator.

GOAL: Your goal is to be able to picture how position, velocity, and acceleration describe the motion of an object. When you're done, you should be able to sketch a graph from a written description of some motion and describe the motion depicted in a graph of position, velocity, or acceleration versus time.

You can click and drag the man around to see how his position, velocity, and acceleration are related to the motion you generate for him. This gets a little messy, though, as smoothly moving the man to get pretty curves is difficult. Instead of mouse-dragging the man, you can use the sliders at the left or type numbers in the fields for the man's position, velocity, and/or acceleration. At the bottom of the screen, ***click the checkboxes*** to show the man's velocity and acceleration vectors (arrows).

After you've gotten a feel for how the simulation works, discuss the questions on the next page with your group. Try different velocity and position combinations to figure them out.

DQ1) What does the "Position" value tell you about the man?

The absolute value of the position tells you how far away from the origin the man is.

What does the +/- sign of the position tell you?

The sign tells you whether his is displaced in the positive or negative direction. Positive is often to the right and negative is often to the left, but you can choose them any way that you like.

DQ2) What does the "Velocity" value tell you about the man?

The absolute value of the velocity tells you how fast the man is currently travelling.

What does the +/- sign of the velocity tell you?

The sign tells you whether he is travelling in the positive (often chosen rightward) or negative (often chosen leftward) direction. You can choose any directions you like for positive or negative directions.

Now simulate the motion for **Motion 1** and **Motion 2** described above and fill in the "Simulation Result" columns. You'll probably want to type the given values directly into the fields at the left. (don't forget to hit "enter" after you type the value in) You also might want to turn the walls off ("Free Range" under the "Special Features" Menu at the top).

Talk together in your group to resolve any differences between your predicted graphs and the simulation results. If there were any differences, summarize them here and explain why the simulation looks different from your prediction.

DQ3) How are the position and velocity graphs related?
(i.e. what about the position graph changes as velocity gets bigger/smaller and +/-?)

As the magnitude of the velocity gets bigger, the position graph gets steeper. Positive velocity gives the position vs. time graph a positive slope. Negative velocity gives the position vs. time graph a negative slope.

In short, the velocity is the slope of the position vs. time graph, i.e. the velocity is the first derivative of the position with respect to time.

DQ4) How are the velocity and acceleration graphs related?

As the magnitude of the acceleration gets bigger, the velocity vs. time graph gets steeper. Positive acceleration gives the velocity vs. time graph a positive slope. Negative acceleration gives the velocity versus time graph a negative slope.

In short, the acceleration is the slope of the velocity vs. time graph, i.e. the velocity is the first derivative of the velocity with respect to time.

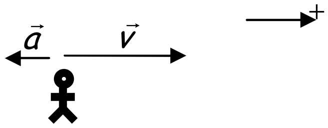
DQ5) Use the simulation to create motion for the man with the following properties. Below each description, sketch a snapshot of the man, his velocity, and his acceleration.

In all of the following pictures, we've chosen "to the right" as the positive direction. This is indicated by the symbol ⟶+ .

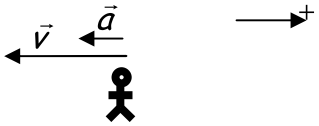
The general idea is that if the velocity and acceleration are in opposite directions, the object is slowing down. If they're in the same direction, the object is speeding up.

Note that the +/- sign of the acceleration does NOT tell you whether something is speeding up or slowing down. It only tells you which way in the acceleration points in space. Speed changes are determined by whether the acceleration points along ("helps") or against ("hurts") the current velocity.

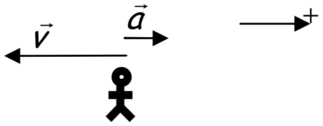
a) The man has a negative acceleration and is slowing down.



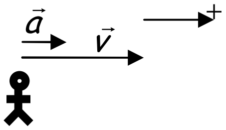
b) The man has a negative acceleration and is speeding up.



c) The man has a positive acceleration and is slowing down.



d) The man has a positive acceleration and is speeding up.



Reflecting Back: What's the difference?

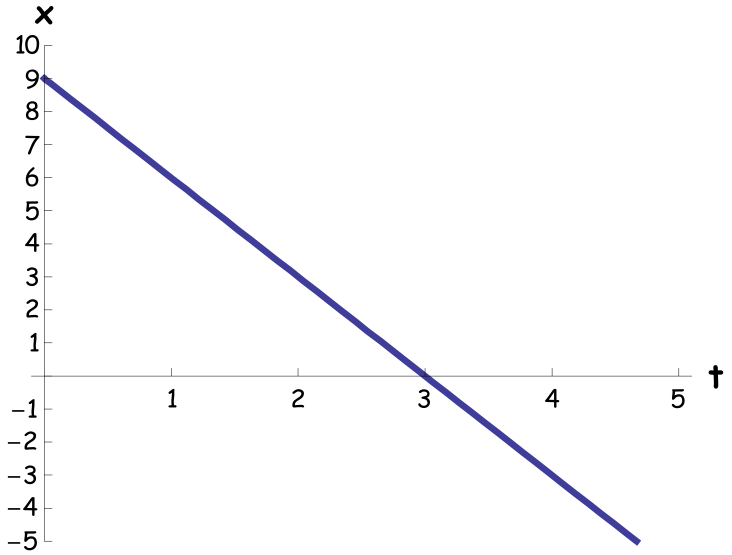
Motion 1) I walk 2.5 m/s eastward for 2 seconds and then 2.5 m/s westward for 2 seconds. All displacements are measured from my starting position.

Motion 2) I accelerate eastward from rest at a constant rate of 2.5 m/s^2 for 2 seconds, and then I accelerate westward at a constant rate of 2.5 m/s^2 for 2 more seconds. All displacements are measured from my starting position.

Why do you think we gave you two questions with nearly identical wording? (What point were we trying to make?)
You were trying to make the point that changing the direction of the acceleration does NOT immediately change the "direction of motion" (the direction of the velocity). The object's instantaneous velocity might slow down instead of speeding up (or vice versa), but you must lose all the speed you had built up before the direction of motion will change (at least in one dimension).

In general, how can the position be determined from the velocity graph?
Velocity is the derivative of the position vs. time graph, so position must be the integral ("anti-derivative") of the velocity. Geometrically, this means the position is the area under the velocity vs. time curve.

DQ6) Talk about each of the pictures below with your group. Try to describe the velocity and acceleration represented by each graph:



a. Graph A

  i. Velocity (positive/zero/[✓ negative])

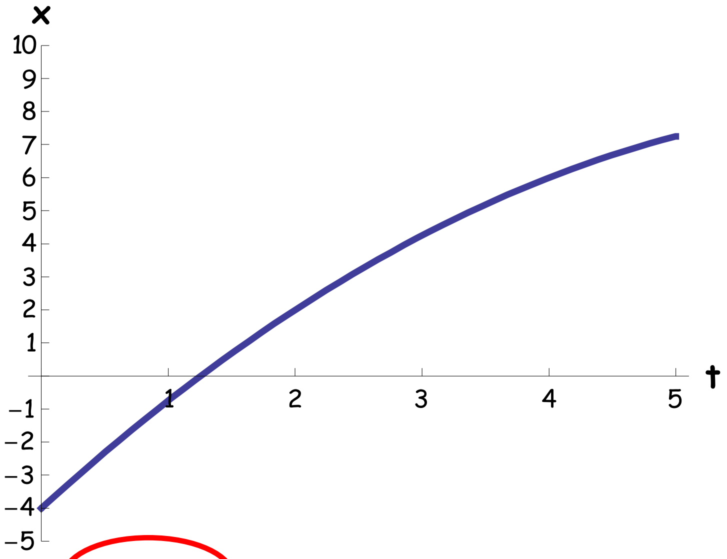
  What aspects of the graph helped you come to this conclusion?
  The position vs. time slope is always negative. Since the velocity value is the slope of position vs. time, the velocity must always be negative.

  ii. Acceleration (positive/[✓ zero]/negative)

  What aspects of the graph helped you come to this conclusion?
  The acceleration is the change in velocity over time. The slope of this position vs. time graph never changes, i.e. this graph is a straight line. Since the slope never changes, the velocity always has the same value. This means the velocity doesn't change. If the velocity doesn't change, then the acceleration is zero.

  An alternative, calculus-based reasoning is a bit quicker. The acceleration is the first derivative of the velocity with respect to time. The velocity is the first derivative of the position with respect to time. This means the acceleration is the *second* derivative of the position vs. time. The second derivative gives the concavity of the graph. Straight lines have zero concavity, therefore the acceleration is zero.

b. Graph B



i. Velocity ([✓ positive]/zero/negative)

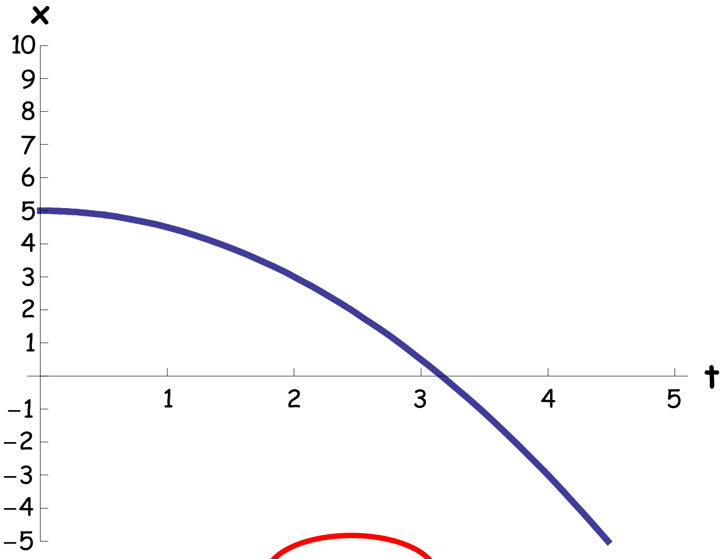
What aspects of the graph helped you come to this conclusion?
This slope of this position vs. time graph is always positive, therefore the velocity is always positive.

ii. Acceleration (positive/zero/[✓ negative])

What aspects of the graph helped you come to this conclusion?
The slope starts steeply positive and becomes shallowly positive. The velocity is therefore less positive (this can also be stated as being "more negative"). A less positive (more negative) velocity over time implies a negative acceleration.

The alternative calculus-based reasoning goes as follows:
This graph is concave down, so it has a negative second derivative. Since the acceleration is the second derivative of position with respect to time, the acceleration must be negative.

c. Graph C



i. Velocity (positive/zero/[✓ negative])

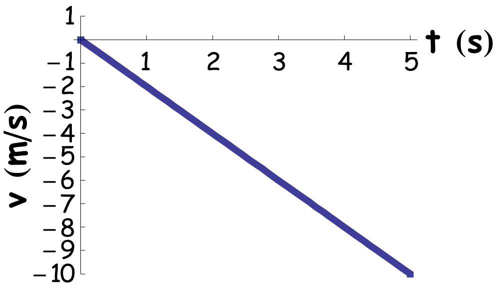
What aspects of the graph helped you come to this conclusion?
The slope of this position vs. time graph is always negative (with the possible exception of zero at t=0). The velocity is therefore always negative.

ii. Acceleration (positive/zero/[✓ negative])

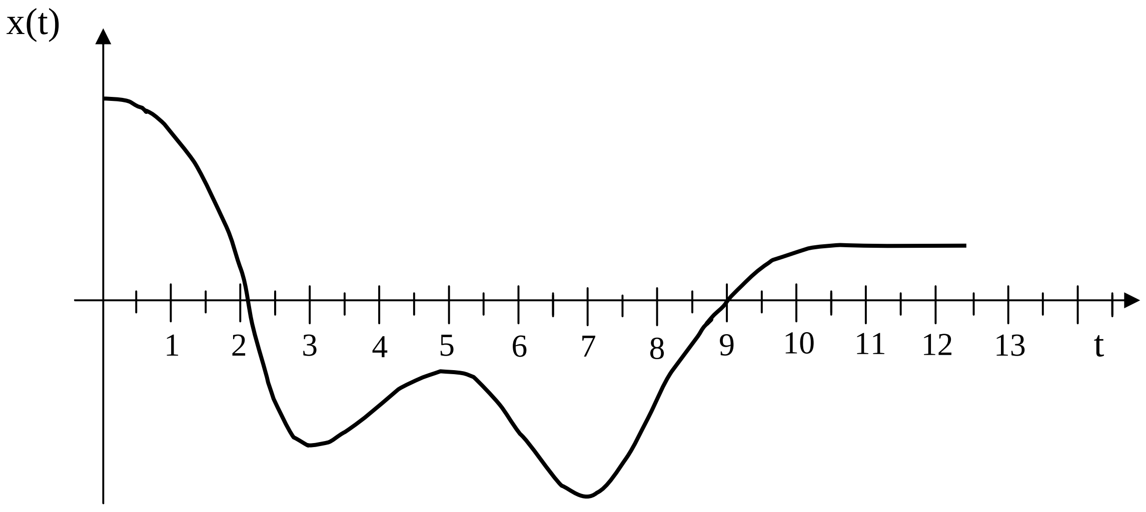
What aspects of the graph helped you come to this conclusion?
The slope begins shallowly negative and become steeply negative at later times. The velocity therefore becomes more negative as time goes on, implying a negative acceleration.

Alternatively, this graph is concave down, implying a negative second derivative. Since the second derivative of position with respect to time is the acceleration, this motion must therefore have a negative acceleration.

iii. Sketch a graph of the velocity of the object as a function of time for Graph C. Is this graph consistent with your verbal description of its motion?
The velocity begins at zero and becomes more negative. The position vs. time graph looks like a parabola. If you've had calculus, you'll remember that the derivative of t^2 (parabola) gives 2t (a line). Mostly, we just need a sketch that starts at v=0 and becomes more negative as time goes on.



DQ7) Given below is a graph that represents an object's horizontal position with respect to time, x(t), where time, t, is measured in seconds.



a) At what times is the object's velocity equal to zero?
Position slope is zero (horizontal) at t=0 s, 3 s, 5 s, 7 s, and 10.5 – 12.5 s. These are the times with zero velocity.

b) During what time intervals is the object's velocity negative?
The position vs. time slope is negative over the intervals 0 – 3 s and 5 – 7 s . These are therefore the time intervals with negative velocity.

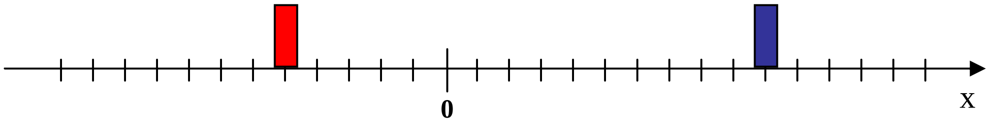
c) In a few sentences, describe the motion of the object during the time interval between t=2 seconds and t = 5 seconds. (Your description should include descriptions of the object's position, velocity and acceleration).
From 2 to 3 seconds, the object starts at a slightly positive position. It travels with a negative velocity while slowing down. It therefore has a positive acceleration.

At 3 seconds, the object (now at a negative position) turns around and travels with a positive velocity, speeding up (i.e. slope gets steeper) until about 4 seconds. It therefore has a positive acceleration over this interval.

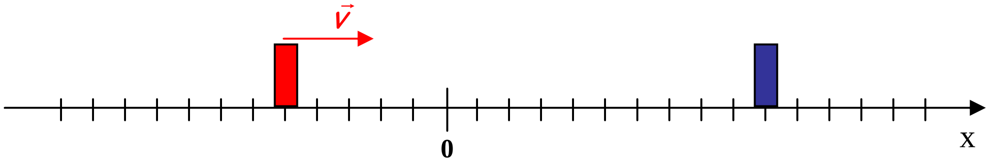
From 4 seconds to 5 seconds, the object (still at negative position) still continues with positive velocity back toward the origin, but slows down. It therefore has a negative acceleration over this period.

d) Drawn below is the object at t = 0. Add to that figure the object at t = 8 seconds. Draw its velocity and acceleration vectors at both times.

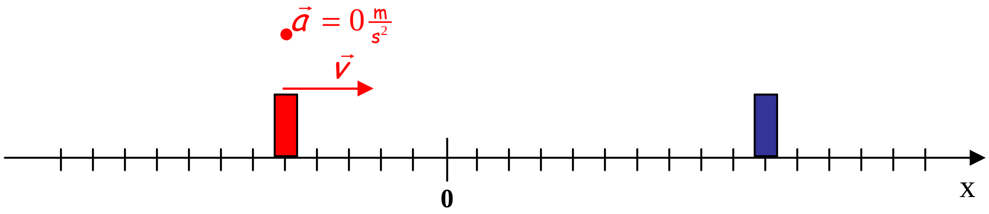
We must first know where to draw the object. The initial position drawn is at 10 m. Looking at the graph at t=8 seconds, the object is about half this distance in the negative direction. We'll therefore draw the object at x=-5 meters:



The velocity at t=8 seconds is positive (positive slope of x vs. t). It therefore points in the positive direction (to the right in this problem).



Looking at the x vs. t graph just before 8 seconds, the slope was smoothly increasing. Looking just after 8 seconds, the slope is smoothly decreasing. Thus, the acceleration goes smoothly from positive to negative at t=8 seconds. Therefore, the acceleration is zero at t=8 seconds. (the calculus way is to realize that t=8 seconds is a point of inflection where the second derivative, and therefore the acceleration, is zero). To draw an acceleration of zero, we can just place a dot (no magnitude) labeled a=0 m/s^2.



DQ8) John and Meg begin driving at the same instant but at different initial positions. John starts off 100 miles to the west of Meg. John drives east with a speed of 65 miles an hour and Meg drives west at a speed of 75 miles an hour. Eventually they pass and see each other for a brief moment. 0.5 hours after this initial passing John turns his car around to try to catch up to Meg. John drives west at a speed of 90 miles an hour until he catches up with Meg.

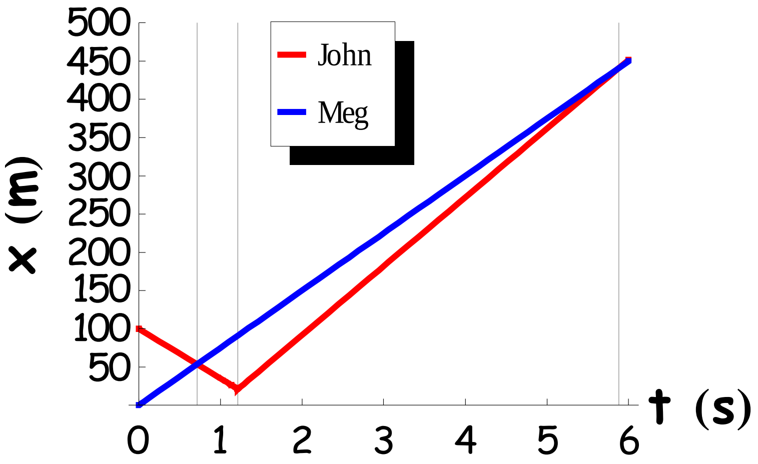
a. Sketch a position versus time graph of the motion of the two drivers from the time when they start driving until they pass for the 2nd time. On your graph make west the positive direction.

We know that John starts out with a 100 mile head start in the westward (positive) direction. Let's use Meg's initial position as the origin.

Meg's travelling quickly in the positive (westward) direction, so she should have a positive, steep slope. John's travelling more slowly in the negative (eastward) direction, so he should start with a negative, shallow slope.

They meet (graphs cross) then continue for another half hour. After that half hour has passed, John's velocity switches direction and gets faster. His graph should therefore switch from shallow negative to steep positive. Eventually, he will catch up to Meg.

The three times mentioned (John catches Meg, John turns around, and John catches Meg again) are highlighted with vertical lines in the graph below.



b. How long after they initially start driving do John and Meg pass one another? (Give the times for both passings).

We'll start by finding the first time at which John and Meg meet. Then we'll see what their separation is half an hour later. We can then solve for the time it takes John to make up this new distance at 90 miles/hr. To get the last meeting time, we'll add up all three calculated time intervals to get the time from the starting point.

1) Find the time when John and Meg first meet.

Symbols used:

d_J = John's initial westward distance

v_{J1} = John's initial speed

v_M = Meg's speed throughout the problem

t_1 = the time when John and Meg first meet

Write John and Meg's positions as functions of time.	$x_J(t) = d_J - v_{J1}t$ $x_M(t) = v_M t$
When John and Meg meet, their positions are the same. We therefore need to find the time at which these positions are equal.	$x_M(t_1) = x_J(t_1)$ $v_M t_1 = d_J - v_{J1}t_1$ $t_1(v_M + v_{J1}) = d_J$ $t_1 = \frac{d_J}{v_M + v_{J1}} = \frac{100 \text{ miles}}{65\frac{\text{miles}}{\text{hr}} + 75\frac{\text{miles}}{\text{hr}}}$ $= \boxed{0.714 \text{ hours}}$

2) Find the separation between John and Meg after the next half hour.

Symbols used: d_2 = separation between John and Meg 0.5 hrs after they meet

Since John and Meg start at the same position for this part of the motion, we can just use their velocities multiplied by their times to get their displacements for this part of the motion.

Find the displacement for John and Meg using their first meeting position as the origin.	$x_J(0.5\text{ hr}) = \left(-65\frac{\text{miles}}{\text{hr}}\right)(0.5\text{ hr}) = -32.5\text{ miles}$ $x_M(0.5\text{ hr}) = \left(+75\frac{\text{miles}}{\text{hr}}\right)(0.5\text{ hr}) = 37.5\text{ miles}$
The total separation is the difference in their displacements (i.e. where Meg is minus where John is).	$d_2 = x_M(0.5\text{ hr}) - x_J(0.5\text{ hr})$ $= 37.5\text{ miles} - (-32.5\text{ miles})$ $= 70\text{ miles}$

3) Find the time it takes John to make up this new separation at 90 miles/hr.

Symbols used:

$d_2 =$ Meg's westward distance after the half hour

$v_{J2} =$ John's new speed (i.e. $90\ \frac{\text{miles}}{\text{hr}}$)

$v_M =$ Meg's speed throughout the problem

$t_2 =$ the time (after the half hour) for John to make up the new distance d_2

Write John and Meg's positions as functions of time. Note that this time Meg has the westward head start and both are moving westward.	$x_J(t) = v_{J2}t$ $x_M(t) = d_2 + v_M t$
When John and Meg meet, their positions are the same. We therefore need to find the time at which their positions are equal.	$x_J(t_2) = x_M(t_2)$ $v_{J2}t_2 = d_2 + v_M t_2$ $t_2(v_{J2} - v_M) = d_2$ $t_1 = \frac{d_2}{v_{J2} - v_M} = \frac{70\text{ miles}}{90\ \frac{\text{miles}}{\text{hr}} - 75\ \frac{\text{miles}}{\text{hr}}}$ $= 4.667\text{ hours}$

4) Add all the time intervals to get the final time at which John and Meg meet.

$$\begin{aligned} t_{tot} &= t_1 + (0.5\text{ hr}) + t_2 \\ &= (0.714\text{ hr}) + (0.5\text{ hr}) + (4.667\text{ hr}) \\ &= \boxed{5.88\text{ hr}} \end{aligned}$$